

SITUATIONS OF PSYCHOLOGICAL COGNITIVE NO-GROWTH

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We present and discuss three classroom situations where failure emerges unexpectedly after initial success and contend that they cannot be sufficiently explained by theories of psychological cognitive growth as surveyed in Tall [2004]. The discussion hinges on the social implication of psychoanalysis as developed by Slavoj Žižek [2002].

INTRODUCTION

Psychoanalysis has made its entrance into Mathematics Education via considerations of affect and cognition. Breen [2004] sought to deal with a case of a student's anxiety through a change in the teacher's attitude. Evans [2004] approaches the relationship of beliefs, emotions and motivations through the study of films that present mathematics as a work of genius. Falcão et al. [2003] discuss affect and cognition approaching the mathematics learner as *possessor of a* subjectivity that is always embedded in culture. Hannula, Maijala and Pehkonen [2004] point out that beliefs related to mathematics (self-confidence) have an influence on students' achievements. Morselli and Furinghetti [2004] consider the connection between cognitive and affective aspects and look for answers in the domain of affect. Walshaw [2004] looks for a conceptualization in Lacan and Foucault that could aid the interpretation of subjectivity. Cabral [2004], Cabral and Baldino [2004], Carvalho and Cabral, [2003] assume a Lacanian perspective and introduce the concept of pedagogical transference. The importance of framing cognition in a wider sociological frame has been demonstrated in PME28 whose main theme was "inclusion and diversity" [Gates, 2004; Johnsen Høines, 2004].

In this paper, we take advantage of another slant of Lacanian Psychoanalysis that has been developed by Slavoj Žižek [2002] and leads to the analysis of social ideological formations. We contend that there is in cognition something more than cognition itself and that, in order to apprehend this surplus, theories of psychological cognitive growth do not suffice. We make an exercise of Hegel's dialectics on Tall's [2004] survey of theories on *psychological cognitive growth* in order to show that these theories have a built-in social exclusion bias. Then we present three episodes of what we call *no-growth situations* that, as such, escape the appreciation of cognitive growth theories. We interpret these situations eliciting their implicit discourse which has the form of present day ideologies: "Yes, I know, but nevertheless...". "I know that school knowledge is important but nevertheless..." Our final discussion relates cognition to three forms of school authority that students, teachers and mathematics educators corroborate in order to disavow (the feeling of) castration: the institutional, the manipulative and the totalitarian forms. It will not be very pleasant to find

ourselves as mathematics educators implicated in the support of such forms of authority, but perhaps this is the unbearable dimension of the “P” in “PME”.

THE EXCLUSION BIAS OF COGNITIVE THEORIES

Tall [2004] seeks to dress an overall universal picture of PME meetings from the point of view of individual psychological cognitive growth. He makes a comprehensive survey of Piaget’s empirical, pseudo-empirical and reflective abstractions, Bruner’s enactive, iconic and symbolic representations, Fishbien’s intuitions, algorithms and formal aspects of mathematical thinking, Skemp’s perception, action and reflection types of activity, Van Hiele’s levels, Dubinsky’s APOS theory, Sfard’s operational operational/structural theory, Lakoff’s embodiment of thinking in biological activity. Grounded on the interplay of these theories, Tall attempts a synthesis intended to encompass the developments from conception to mature man and from discalculic children to research mathematicians. He arrives at “three worlds” into which cognitive growth can be categorized: the worlds of perception, of symbols and of properties. “Different individuals take very different journeys through the three worlds” he says [ibid: 285].

The reader is a little deceived since, instead of a synthesis, one could expect a global appreciation of such theories so that they could be sublated (*afhoben*) towards something new. After all, their similarities are much more striking than their differences. Why are there so many theories focusing on the same object, namely, psychological cognitive growth? Besides, they do not stem from an effort to make sense of a large amount of empirical data; on the contrary, they rely more or less heavily on their respective authors’ introspection. Experiences and studies tend to confirm, infirm or answer specific questions put by the theory, rather than to discover and tackle new phenomena.

From a philosophical point of view, the general idea of growth implies a change in magnitude while a certain basic entity keeps its identity invariable: the “individual” who transits through the “worlds” remains an invariable seat of magnitude. “A magnitude is usually defined as that which can be increased or diminished” [Hegel, 1998:186]. Hegel shows that this is a circular definition: “magnitude is that of which the magnitude can be altered” [ibid] but instead of discarding the definition as we would do in mathematics, he takes it up as the starting point of the very Notion of magnitude. Indeed, the definition has the merit of pointing out the external agent, the *author*, who first thought of it as a reasonable one. It is the author who provides the invariable background against which growth can be thought.

In so far as theories of psychological cognitive growth refer to mathematics, they rely on a scale of values based on mathematical knowledge itself, a hierarchy rising from numerical pre-linguistic to the axiomatic and formal. Their authors speak from the position of one who has reached the apex of the stages or levels of their scales. They focus on psychological cognitive growth from the perspective of an *autonomous ego* hovering over the changes of magnitude of others, out of reach of any criticism.

Considering the transformation of quantity into quality, Hegel warns us that a field that gets too wet ceases to be a field and becomes a swamp. At what precise amount of humidity did it become a swamp? Which hair thread one has to lose in order to be considered bald? At what precise moment a graduate student becomes a research mathematician? At what precise moment a child surpasses its discalculic condition? These are symbolic determinations and as such they are intrinsically retroactive: once they are verified it is found out that the new situation they constitute existed a little before. Why? Essentially an external agent is responsible for the declaration of the new state of affairs. In order to be able to think changes of levels or states simply as “growth”, one has to abstract from the external social agent who attributes different magnitudes to an identical subtract. The identity resumes to the external social agent.

Leaving their authors out, cognitive-growth theories assume the status of scientific subject-free theoretical speeches. This effort leads to an absolute scale of values in which all subjects are positioned, the author occupying the apex. The tendency is almost unavoidable to pass from “growth” to “lack”, “deficiency”, “shortage”, etc. This is the perverse social effect of cognitive theories. We do not claim that a further effort should be made towards a “perfect theory” that would be politically neutral. These theories represent an important logical moment. The contribution of psychoanalysis goes in the opposite direction: simply, the wills and desires of the authors must be brought to the fore. This is what we intend to do below.

THREE NO-GROWTH SITUATIONS

The episodes below were extracted from classes of two freshmen courses, one in Analytic Geometry (AG) the other on calculus (C1) given in August-December 2004 for repeaters in the engineering program of our institution. Ten students enrolled in AG, six concludes the course and four passed; twelve enrolled in C1, seven concluded and two passed. Only one student of each course was not enrolled in the other. Classes met during four consecutive 50-minutes periods on Tuesdays (AG) and Thursdays (C1) totalizing 60 periods for each course. The text book was Stewart [1999] chapter 13 for AG and chapters 1 to 4 for C1. Classes had a tutorial format assisting individuals or couples of students. Each class ended with a 40-minutes hand-in individual exercise, graded and returned to the student’s scrutiny in the beginning of the following class. Very seldom students took photocopies of graded exercises. These exercises made 40% of the passing grade the other 60% came from two mid-terms and one final open-book written individual exams. Classes started with a proposition of exercises to be worked out. Students could never do more than two or three exercises per day. Pedagogical remarks stressing important points were inserted at each class as difficulties arose.

Episode 1: Mary

Mary had been our student in a high school course on elementary algebra. The only way she could solve algebraic equations was by trial and error. She entered the university, failed AG and C1 and became our student in the described environment.

In Agu-24, the exercise was: “Given points in the plane A , B , C and D , find x and y such that $AD = x AB + y AC$ ”. Mary found the system of equations, tried to solve it by substitution but made a mistake:

$(3x-12)/2 = 3x-6$. The class of Sep-9 was dedicated to solving algebraic equations by the “method of transformations”: 1) operate simultaneously on both members; 2) replace one member by an equal one. Mary showed some proficiency but made the same mistake again: $(7+14x)/x = 21$. In the class of Sep-21 we made sure that all students could solve systems of two and three equations by Cramer’s rule. In Oct-10, one question of the mid-term exam was: Draw the straight lines $r(t) = (5, 6) + t\langle 1, 3 \rangle$ and $y = 8 - 3x$, write the first one in reduced form and determine their intersection up to three decimal places. Mary solved the system by substitution and this time she got it right. Would we say success?

Mary passed AG but not C1. One of the questions of the second-chance C1 final exam in Dec-21 was: Find the intersection of the tangent line to $f(x) = x^2 + \frac{1}{x}$ at $x = \frac{1}{2}$ with the secant line through $x=1$ and $x=2$. Mary arrived at the system (with one wrong coefficient) and got stuck.

Mary: Where can I find “intersection of straight lines” in the book?

We showed her the topics of intersection of lines and planes, of two lines in space and the statement of the question in the mid-term exam reminding her that she had got it right. She did not have a copy of the exam with her and her classroom work with a similar question was incomplete. When she finally handed her paper in with the question blank, we checked what sense she made of lines and equations. We drew two lines with their equations $y = 2x - 3$, $y = -3x + 5$. She indicated the correspondence of x and y in the equation and points in the plane.

Teacher: (Pointing at the intersection): What happens at this point? What are the values of x and y ?

She recognized that the same x and the same y should fit into both equations. We insisted:

Teacher: How can you find this x and this y ? (She remained silent, looking at the picture.)

Teacher: Are you making trials?

Mary: Yes.

Resume: After one semester of intense tutoring work Mary reinforced her confidence in algebraic transformations and was able to solve a system of two equations by substitution. Yet, at the crucial pass/fail moment of the exam, she went back to her old high school strategy of trial and error.

Episode 2: John

In Dec-12 John was able to correctly solve the items below in the final exam.

$$d\left(\frac{\tan(3x)+4}{\sin(3x^2+4x)}\right)=\dots, \frac{d}{dx}\left(\sqrt[5]{32}e^{3x+4}\right)=\dots, \sec(10x)\tan(10x)dx=d(\dots), \lim_{x \rightarrow 0} \frac{x^2}{e^x - x - 1} = \dots$$

He did not get a passing grade and had to take the second-chance final. In the question of finding the minimum distance of a point to a curve, he repeated the mistake that we had pointed out in his exam one week before: the derivative of $(x^2 + 3x - 5)^2$ was simply $2(x^2 + 3x - 5)$ and in the question of related rates he differentiated $3 \cos \theta = z$ as $3 - \sin \theta = dz$.

Resume: After one semester of tutoring John could show proficiency in applying the chain rule to rather involved composition of functions. However, at the final moment he seemed to have forgotten all and scribbled absurd equalities.

Episode 3: Students

During the first two weeks (Aug-19, 26) of C1 we made sure that all students could perform graphical exercises on derivatives and primitives reasonably well. Given the graph of an arbitrary function, draw tangent lines at several points, evaluate the slopes, plot the slopes as the graph of the derivative and conversely, starting from a given graph, interpret the ordinates as the slopes of a primitive and draw its graph through a given initial point. A protractor graduated in tangents was provided. The derivative was introduced as the “name” given to the slope of the tangent line and we made sure that every student could explain the meaning of this definition. Discussions of the relation of increasing/decreasing functions with the signs of derivatives were provided. In the next weeks we worked on algebraic equations (Sep-02), rules of differentiation (Sep-09), derivatives of elementary functions via limits (Sep-16) and graphs of cubics (Sep-23). Finally we came to optimization problems (Sep-30). Students were asked to read the first example in the text book. At a certain point they read: “So the function that we wish to maximize is $A(x) = 2400x - 2x^2$ $0 \leq x \leq 1200$ ” [Stewart:278]. They had no problems so far. “The derivative is $A'(x) = 2400 - 4x$, so to find the critical numbers we solve the equation $2400 - 4x = 0$ ” [ibid]. At this point the six students in class asked “Why?”

Teacher: Well, if you have a function like this (drawing a graph with a local maximum) how much do you think that the derivative will be at this point?

Students: I don't know.

Teacher: The derivative is the name of what?

Students: (After some help for recollection): It is the slope of the tangent line.

This seemed to suffice for two of the students but the other four still could not make any sense.

Teacher: (Showing a tangent line just a little to the left of the maximum): Is the slope of this line positive or negative? (The strategy was to move the tangent to the right until it reached the point of maximum.)

Students: I don't know. What do you mean by "slope"?

The exercises of the first two weeks had had to be retaken before they could express any connection between extreme points and derivatives. This took most of the day.

Resume: Everything that they had learned in the first two weeks about slopes and tangents was not available any more.

DISCUSSION

We presented a picture where the natural outcome would point towards growth and in many reports could be held as a bulletin of victory. Mary learned how to solve systems of equations by substitution and abandoned her empirical trial and error strategy; John proficiently learned the chain rule and all students could reasonably perform graphical correspondences between derivatives and primitives. However we went one step further and checked this success in the day after. It had fallen into a black hole! No-growth situations mean *success followed by unexpected failure*.

A new notion such as *no-growth situations* naturally faces criticism. Is it necessary? Do these situations exist at all? Arguments may contend that we did not provide enough data in support of our concept: how was the affective teacher student relation? Were the student's mistakes discussed in class? What sort of extra-class help was provided? Did the students have the necessary requisites to take a calculus course? An endless list of extra data may be required postponing the decision indefinitely or until a point is reached where the reported no-growth situation may be characterized as *failed-growth*: had the teacher behaved more friendly, had the method been adequately applied, had this or that been different, then *growth* could have occurred. True, the reported situations can be considered a peripheral problem in cognitive growth theories; we prefer to take them as a *central problem* in a new way of looking at "growth". Should we call this new look "social cognition"?

We argue that it is important but not sufficient to focus on growth when it occurs. We have to crucially consider *what the student does overnight with what he has learned during the day*, that is, what he does *outside* the school. Every day the students in the reported situations confirmed their will of becoming good professional engineers and behaved accordingly, coming to class and working hard on the exercises. However, from one day to the next they treated their learning in a way as to deny such good intentions. In our interpretation their implicit overnight discourse could be:

Mary: 'I know that my trial and error method to solve equations falls short of the course needs and I have learned other methods; nevertheless trial and error it is *my* method, my deep personal enjoyment and I will stick to it.'

John: 'I know how to operate differentials according to the strict chain rules as I have learned in this course; nevertheless I will do according to my former understanding: squares are replaced by twice the thing and cosine by minus sinus.'

Students: 'We know that what we learn in one class will be necessary for the next one; nevertheless we do not take the trouble of keeping our learning under account.'

According to such interpretations (there may be others) the reported no-growth situations may be referred to one of the three elementary structures of the exercise of authority which function socially as three modes of disavowing castration.

Traditional authority is based on what we could call the *mystique of the Institution*. Authority bases its charismatic power on symbolic ritual, on the form of the institution as such. (...) Socrates' argument could thus actually be linked to the phrase 'I know, but nevertheless...': 'I know that the verdict that condemned me to death is faulty, but nevertheless we must respect the form of the law as such'. [Zizek, 2002:249]

'I know that the value of school knowledge is questionable and that I will have to undergo training in my first job; nevertheless I believe that this knowledge represents the distinctive herald of my social group and I must endeavor to acquire it. The Emperor wears fine clothes because he is the Emperor.' The interpretations we gave of the students' overnight speeches certainly do not support this form of authority.

The second mode corresponds to what might be called *manipulative authority*: authority which is no longer based on the mystique of the institution – on the performative power of symbolic ritual – but directly on the manipulation of its subjects. This kind of logic corresponds to a late-bourgeois society of 'pathological Narcissism' (...) constituted of individuals who take part in the social game externally, without 'internal identifications' – they 'wear social masks', 'play their roles', not taking them seriously'. (...) The social role of the mask is directly experienced as a manipulative imposture; the whole aim of the mask is to make an impression on the other. [Zizek, 2002:251].

'The social role of the school institution is directly experienced as a manipulative imposture; its whole aim is to make an impression on the other, school knowledge is useless, only the certificate counts.' Would peripheral Third-World countries typify the "late bourgeois societies" mentioned by Zizek? These countries have received the "masks" of neo-liberalism, of globalization, of free trade, of international help and loans as impostures leading to increased exploitation. It is not surprising that such an understanding reflects itself in school, splitting *knowledge* and *belief*: 'yes I know that the Emperor wears fine clothes, nevertheless I believe he is naked and I act accordingly'.

The third mode, fetishism *stricto sensu*, would be the matrix of *totalitarian authority*. (...) The totalitarian too does not believe in the symbolic fiction in his version of the Emperor's clothes. He knows very well that the Emperor is naked (...). Yet in contrast to the traditional authority, what he adds is not "but nevertheless" but "just because": *just because* the Emperor is naked we must hold together the more, work for the Good, our cause is all the more necessary. [Zizek, 2002:252].

'We know very well that imparting upper class central countries knowledge such as mathematics, to proletarian students of peripheral Third World countries is impossible, that raising the economy of a country through education is a hopeless dream, that all the efforts in favor of Mathematics Education have had a proportionally pale effect. Just because we know, since Freud, that education is one

of the four impossible endeavours, Mathematics Education is the more necessary. Commitment to it is our charming mode of disavowing castration.'

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