EXPLORING HOW POWER IS ENACTED IN SMALL GROUPS

Mary Barnes

University of Melbourne

This paper presents an analysis of the enactment of power during group discussions in high school mathematics. The class studied was working on introductory calculus using a collaborative learning approach. In analysing a group discussion, I first traced the flow of ideas, looking at when and by whom a new idea was introduced, and how others responded. I next divided the transcript into "negotiative events" and looked at how transitions from one event to the next came about. These analyses made it clear that some students had more power than others to influence the course of the discussion, but that this was not related to their mathematical capabilities.

INTRODUCTION

The research reported here is part of a larger study of student-student interactions during collaborative learning in mathematics (Barnes, 2003) conducted in classrooms where students worked in small groups, with shared goals, on challenging unfamiliar tasks. They were not taught standard solution procedures in advance, but were encouraged to construct new concepts by recalling prior knowledge and combining and applying it in new ways. In whole-class discussions following the group work, students explained solutions, asked questions, and shared insights, and the class tried to reach a consensus. Collaborative learning is encouraged by recent mathematics curriculum documents that emphasize the importance of fostering communication skills and encouraging mathematical dialogue (e.g., AAMT, 2002). Collaborative learning is not always successful, however. This paper explores ways in which social interactions within collaborative groups can interfere with the learning process.

THEORETICAL FRAMEWORK AND LITERATURE REVIEW

The theoretical perspective underlying the study is that of sociocultural theory (see Davydov, 1995; Lerman, 2001). Based on the work of Vygotsky, sociocultural theory asserts that all learning is inherently social, resulting from the internalisation of processes developed in interaction with others. In addition, the theory claims that learning is mediated by signs and cultural tools, including language (both oral and written), symbols, gestures and artefacts. This means that studies of small-group learning need to attend not only to spoken discourse, but also to the participants' body-language, tone of voice, direction of attention, and the artefacts they are using.

Recent research on collaborative learning has studied the interactions within collaborating groups. Most of this has focussed on cognitive and metacognitive aspects of the interactions (e.g., Forster & Taylor, 1999; Goos, Galbraith & Renshaw, 2002), but I believe that social aspects need to be considered also, because poor communication and social relationships within a group can result in failure to engage fully with the task, or can limit the range of solution pathways considered. For

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collaboration to be effective, appropriate socio-mathematical norms (Yackel & Cobb, 1996) need to be established. These include expectations that everyone will contribute, that others will attend to what is said, and that assertions will be justified.

Cohen (1997) describes status inequalities as a cause of unequal interaction within groups, resulting in unequal learning opportunities. Factors that determine a student's status include perceived ability, popularity with peers, as well as gender, social class and ethnicity. Cohen draws on Expectation States Theory to explain how a student's status sets up performance expectations that can be resistant to change. Cohen and her colleagues used mainly quantitative methods to study inequalities in interactions within groups. My research question was to find ways of using qualitative techniques to investigate how power is enacted, and unequal interaction patterns come about.

THE STUDY

My research was a multi-site case study of classes engaging in collaborative learning, using video to capture classroom interactions. During group work, the camera focussed on one group, and a desk microphone captured their speech. Additional data included interviews with teachers and selected students. This paper focuses on a class of Year 10 students who were following an accelerated mathematics curriculum. The lesson described took place near the end of a sequence on introductory calculus. The class had already investigated gradients of curves, discussed limits, and worked out rules for differentiating polynomials, and how to use calculus in curve sketching. Up to this point, calculus had been presented in an abstract mathematical context, with no discussion of potential applications. The following problem was then presented:

> You have a sheet of cardboard with dimensions 20 cm by 12 cm. You cut equivalent squares out of each corner and fold up the sides to form a box without a lid. What should be the length of the sides of the squares cut out for the box to have maximum volume?

This is a standard problem found in most calculus textbooks, but to these students it was a true investigative task. They had no prior experience of similar problems and no idea of how to proceed. They were not even sure if it was related to their work on calculus, and the teacher gave no hints. There are many possible ways of tackling the problem, with and without calculus. I chose this lesson for detailed analysis, not because it was "typical" in any sense, but because of the contrasting personalities in the group and the complexity of the discussion. This revealed interesting group dynamics which helped to cast light on how power is enacted within small groups.

Introducing the group

During the small-group discussion part of the lesson the camera was focussed on four students, whom I call Vic, Zoe, Charles and Selena. Like everyone in this accelerated class, they were high achievers in mathematics. Vic was a champion athlete, held an elected leadership position within the student body, played in the school band and was popular and confident. He seemed, however, to have a short attention-span and to crave attention. Zoe too was popular and confident, and generally very articulate.

She spoke up frequently in class discussions. In contrast, Charles was awkward, shy and diffident. He appeared to be a loner, with no friends in the class. The teacher commented in an interview on his poor social and communication skills, adding that he was "very bright, a critic". Finally Selena, a new student, and the only class member of Asian background, was shy but eager to be accepted. Other students were unaware of her mathematical thinking capabilities, but did know that some topics which they had studied had not been covered at her previous school, so they may have tended to assume that in general she knew less than they did.

A brief outline of the discussion

During the lesson, the group worked for 35 minutes, following a tortuous solution path that involved many false leads and dead ends. But by the end of the time they had solved the problem by two different methods, one of which used calculus.

They began by trying to make sense of the problem. Although "maximum volume" was stated clearly, Selena and Zoe interpreted it as asking for maximum *base area*, and discussed how small an edge they could turn up and still call the result a box. Selena talked about turning up an edge "as close as possible" to zero, and speculated whether limits were relevant to the problem. Eventually Zoe grasped that the problem was about volume, not area, and claimed that they were now on the right track.

Charles suggested that they let the side of the square cut out be x, and find a formula for the volume in terms of x. Zoe agreed at first, but then abandoned this approach for what she thought was a simpler way and Vic supported her. Selena pointed out a flaw in their reasoning, and the group finally agreed on an expression for the area of the base. After some digressions, Charles prompted them to write the volume as a cubic polynomial, and suggested graphing it (see first transcript below). The others did not think a graph would help, but Selena began to draw the graph on her graphics calculator. Charles explained that a graph would tell them which value of x gave the greatest volume. Zoe ignored this, and proposed asking Miss James if they were on the right track. Miss James first asked them to explain what they had done, followed this with questions like "What are you going to do next?", and then left them.

Zoe invited ideas about what to do, and Selena asked, hesitantly, if they should "do the derivative". Zoe could not see how it would help. Charles supported Selena, and explained why (see second transcript, below). Vic grasped part of what Charles said (about the graph showing where the maximum lay, but not about using the derivative) and acted on it, using a graphics calculator to find the x-coordinate of the maximum turning point. Again, Zoe sidetracked them with the seemingly pointless suggestion of equating the volume to zero, but this eventually led them to conclude that x was between 0 and 6. After an unnecessary substitution to find the greatset volume (not realising that they could read it off the graph) they substituted values of x on either side of their answer to verify that it was indeed a maximum, and announced that they had "done it". The teacher prompted them to explain what they had done, and asked if they could think of another way to solve it, and if they could justify their result.

Selena suggested using the derivative to find the turning point, Charles supported her, (see lines 369-375 below) and Zoe agreed. When they equated the derivative to zero to find the turning points, they struggled for a long time to factorise the resulting quadratic equation. Selena suggested using the quadratic formula, but Zoe and Vic resisted and continued trying to factorise. Eventually Charles concluded that they would have to use the formula, Vic agreed, and he and Selena did the calculation, obtaining the same answer as by the graphical method. As they were explaining to the teacher what they had done, the bell rang bringing the lesson to an end.

ANALYSIS

The complexity of both the range of ideas discussed and the interactions among the students made the transcript difficult to follow and interpret. It was necessary to find methods of data reduction that would help to make visible the phenomena of interest: the interplay between mathematical ideas and the interactions among the students.

Identifying the ideas involved

A first step was to list the different ideas the group discussed, including those that were helpful, and those that proved to be 'red herrings' that led the group astray. I list here the helpful ideas. For reasons of space the 'red herrings' are omitted.

Ideas which helped the group move forward towards a solution:

- Introduce x for the length of the sides of the squares cut out, and find an expression for the area of the base of the box and hence its volume.
- Graph the volume function and, from the graph, find where it is greatest.
- The value of x must be between 0 and 6.
- Substitute the x-value of the maximum point into the volume function to find the greatest volume (only necessary because they did not recognise that the y in their graph represented the volume).
- Check function values on either side of this to verify that it is a maximum.
- Find the derivative and equate it to zero to find turning points.
- Factorise the expression for the derivative to find its zeroes.
- (When factorising proved impossible) Use the quadratic formula.

Tracing the flow of ideas

From the transcript, it was possible to trace the way in which an idea was introduced by one group member, accepted or rejected by others, and perhaps reintroduced later, maybe more than once. To illustrate, I use the idea of *graphing the volume function*. (*Note*: A key to the symbols used in the transcript is given at the end of the paper.)

First mention of idea: Having introduced x to represent the length of the side of the corner square, the group (with some difficulty) found an expression for the volume of the box. They were then unsure what to do. After a short silence, Charles spoke:

298. Chas: Perhaps we should graph it.

299. [8 sec pause. The girls sit back. They seem to be thinking. Vic moves as if stretching his neck. Charles glances a little anxiously in Vic's direction.]

300. Zoe: Wait a minute … Um, okay … Hang on, that was this time … We have to find the limit when X equals zero, maybe /

301. Sel: /How does the graph help it?

302. Zoe: I don't think it does. Oh it might.

303. Sel: Hang on, I'll just see / [begins to draw the graph on her calculator]

304. Vic: /Not in particular, what does it do? It just gives you two points on the axis.

305. [Vic turns round to watch what other groups are doing for 24 seconds.]

306. Zoe: Mm. Well, what we're trying to do is, we're trying to find the value across here. [Points to her diagram] We have to find that.

307. Sel: Um [Uses graphics calculator, murmuring to herself as she presses keys]

308. Chas: Well, that's /

309. Sel: /the graph /

310. Chas: /what value of X gives us the most volume.

311. Chas: [Selena holds out her calculator to Charles.] Is there a turning point there?

312. Sel: Yeah. Two. Um, yeah two.

313. Chas: Yeah, one of them's down there /

314. Zoe: /Shall we ask Miss James if we're on the right track?

315. Sel: Yeah. [Vic has turned back to the group again. He nods.]

Summary 1: Charles' suggestion initially met with no response. Then Zoe expressed doubt and proposed an alternative, based on a misconception. Selena questioned the idea. Vic was dismissive. Then Selena began to draw the graph on her calculator. She and Charles were making progress when Zoe brought the discussion to an end by suggesting that they talk to the teacher, and everyone but Charles agreed.

Second mention of idea: Zoe called the teacher over to them and spoke for the group, but she did not explain everything they had done, and in particular did not mention graphing. As Miss James turned to go, Charles said "We need to graph this". Miss James did not hear, and Zoe interrupted excitedly to propose an unhelpful idea.

Third mention of idea: They discussed a number of suggestions about what to do. Selena asked if they should use the derivative and Charles, in expressing his support, referred to the graph that Selena had drawn on her calculator:

369. Chas: Basically, what I think here is that this turning point [points to the graph on Selena's calculator] um, at the turning point, that's going to be your maximum value for um /

370. Sel: /which is that? [points to something on the table in front of her, possibly on the worksheet, but exactly what is not visible to the camera]

371. Chas: Yeah. Well, maximum value for X, // to get us

372. Vic: //Obviously, so we've to find the value /

373. Chas: /the maximum volume.

374. Vic: [Picks up Selena's calculator] So, trace

375. Chas: So basically you do need to work out the derivative.

Summary 2: Charles was trying to explain why the maximum turning point would give them the answer. This time Vic listened to him, took in part of what he was saying, and acted on it, but gave no sign that he had heard Charles' final statement.

Overview: In this sequence of excerpts, Charles repeatedly made a suggestion without success. Selena was willing to give it a try, but Zoe and Vic repeatedly rejected or ignored what he said. It was not until Vic endorsed part of Charles' final statement that the whole group focused on drawing a graph and used this to find a solution.

I give a second example in less detail. The idea of *differentiating the volume function* and using the derivative to find turning points was first raised by Selena while they were brainstorming what to do (line, 363, just before the start of the second excerpt above). She expressed it tentatively, as a question: "Are we doing, do we do the derivative in that?" Zoe expressed doubt: "Like, what for?" but Charles supported Selena by explaining why it would help (second excerpt). Vic pre-empted him by beginning to use the Trace function on the calculator. The derivative idea seemed to be forgotten until Miss James asked them to think of alternative ways they could use to solve the problem. Selena hesitantly said, "Use the der- deriva-" (line 623). Zoe interrupted to repeat an idea of her own, but Charles spoke in support of Selena. Zoe suddenly seemed to catch on, exclaiming "Yeah, the derivative. It's the turning point." (line 629) and gesturing to show the shape of the graph. The group then used the derivative to find the maximum turning point and hence the maximum volume.

Overview: Again one student, this time Selena, repeatedly tried to make a point, but it was rejected by the group until Zoe gave it her support.

I carried out a similar analysis for each idea discussed. Of eight helpful ideas, Selena initiated three, Charles three, and Zoe and Vic one each, but none were acted upon unless supported by Zoe or Vic or both. This makes it clear that it was not the potential value of an idea that determined its adoption by the group, but whether or not it was supported by at least one of the two students Zoe and Vic. This insight prompted a more detailed look at how the topic of discussion was determined.

Control of the topic of discussion

Clarke (2001) proposed a way of structuring lesson transcripts by dividing them into episodes and further subdividing episodes into *negotiative events*. I adapted his definition slightly to suit the classes I was observing, and defined a negotiative event to be the smallest unit of conversation involving two or more people *with a consistent topic or goal*. A negotiative event may be an entire episode, consisting of many turns or it may be a single utterance followed by tacit assent by another person.

After subdividing the transcript into negotiative events, I set out to investigate how transitions between events came about. Transitions require the complicity of the group: an utterance does not initiate a new negotiative event unless other group members begin to discuss it, or at least assent to it; nor does a declaration such as "That's done!" necessarily terminate an event, unless other group members agree.

To illustrate, Excerpt 1 is a single negotiative event, initiated when Charles proposed graphing the volume function (line 298) and terminated when Zoe suggested asking Miss James (and Selena and Vic assented). Excerpt 3 shows the end of one negotiative event and the beginning of another. The first (deciding what to do) ended when Vic said "obviously, so we've got to find the value" (line 372). The next event (using the graphics calculator to find the maximum) began when Vic said "So, trace" (line 374). Charles' utterance at line 373 was a continuation of what he had been trying to say in his previous four turns and was ignored by the others.

When the entire discussion had been divided into negotiative events, I analysed who initiated and who terminated each and in what way, and recorded this in a table. These were then counted and the results displayed in another table (see Table 1).

	Zoe	Vic	Selena	Charles
Initiations	16	7	4	3
Terminations	14	9	2	1

Table 1: Negotiative events initiated and terminated by each group member

This clearly shows Zoe's dominance, and the relative lack of influence of Charles and Selena. Vic spent a lot of time talking to other groups, so had less influence than Zoe.

DISCUSSION AND CONCLUSIONS

The results support the findings by Cohen and her colleagues about the effects of inequalities in status on interactions within groups. To determine a student's status in the classroom, Cohen (1997) used a combination of peer status (i.e., popularity) and academic status, measured by asking students to nominate who in the class were best at the subject. If such an instrument had been used, it is clear that both Vic and Zoe would have been assigned high status. Both were popular in the class and contributed often to class and small group discussions. In contrast, Charles would have had low status. He was unpopular and inarticulate. The teacher recognised him as "bright" but poor writing skills meant that he did not get high grades in assignments, so it is unlikely that other students would have recognised the quality of his thinking. Selena was new to the class, so had not had enough time become popular, and there was little evidence on which other students could form judgements about her academic ability. Thus, at the time of the study, she too would have had a low status. My analysis has shown that high status students influenced the discussion in the following ways: their ideas (useful or otherwise) were more likely to be accepted by the group; and on most occasions they determined what the group would discuss next. Both of the low status students put forward good ideas, but these were only accepted when endorsed by a high status student. And they had very little opportunity to influence the course of the discussion. By making more transparent the mechanisms by which students establish dominance within a group, this study may help in planning instructional strategies designed to reduce inequities in the classroom and enhance learning for all students.

Cohen and her colleagues identified inequality in participation by counting the number of turns for each student. Looking instead at whose ideas were accepted or rejected, and who determined the topic of discussion, provides a more detailed and more powerful picture of the ways in which power is enacted within small groups.

Finally, a methodological point: tracing the flow of ideas is an innovative approach to analysing complex discussions, as is studying the structure of a discussion to identify how transitions from one topic to another come about. These potentially have wider applications, for example in studying whole-class teaching, or discussions of other kinds, especially in situations where the enactment of power is at issue.

Note 1

Key to symbols used in transcripts:

/	no noticeable pause between turns, along with indications that the first turn was incomplete
//	marks the beginning of overlapping speech
…	a brief pause of 3 seconds or less. (For longer pauses, duration is stated.)
[text]	descriptions of actions, body language facial expressions or tone of voice.

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