

# VYGOTSKY'S THEORY OF CONCEPT FORMATION AND MATHEMATICS EDUCATION

Margot Berger

University of Witwatersrand

*I argue that Vygotsky's theory of concept formation (1986) is a powerful framework within which to explore how an individual at university level constructs a new mathematical concept. In particular, this theory is able to bridge the divide between an individual's mathematical knowledge and the body of socially sanctioned mathematical knowledge. It can also be used to explain how idiosyncratic usages of mathematical signs by students (particularly when just introduced to a new mathematical object) get transformed into mathematically acceptable usages and it can be used to elucidate the link between usages of mathematical signs and the attainment of meaningful mathematical concepts by an individual.*

## INTRODUCTION

The issue of how an individual makes personal meaning of a mathematical object presented in the form of a definition is particularly relevant to the study of advanced mathematical thinking. In this domain, the learner is frequently expected to construct the properties of the object from the definition (Tall, 1995). In many instances neither diagrams nor exemplars of the mathematical object are presented alongside the definition; initial access to the mathematical object is through the various signs (such as words and symbols) of the definition.

In this talk, I argue that Vygotsky's theory of concept formation (1986) provides an appropriate framework within which to explore the above issue of concept formation. Specifically I claim that this framework has constructs and notions well-suited to an explication of the links between the individual's concept construction and socially sanctioned mathematical knowledge. Also the framework is apposite to an examination of how the individual relates to and gives meaning to the signs (such as symbols and words) of the mathematical definition.

## BACKGROUND

Several mathematics education researchers have considered how an individual, at university level, constructs a mathematics concept and some have developed significant theories in response. The most influential of these theories focus on the transformation of a process into an object (for example, Tall, 1995; Dubinsky, 1991; Czarnocha et al, 1999).

According to Tall et al. (2000), the idea of a process-object duality originated in the 1950's in the work of Piaget who spoke of how "actions and operations become thematized objects of thought or assimilation" (cited in Tall et al, 2000: 1).

In adopting a neo–Piagetian perspective, these researchers and their various followers successfully extend Piaget’s work regarding elementary mathematics to advanced mathematical thinking. For example, Czarnocha et al. (1999) theorise that in order to understand a mathematical concept, the learner needs to move between different stages. She has to manipulate previously constructed objects to form actions. “Actions are then interiorised to form processes which are then encapsulated to form objects” (1999: 98). Processes and objects are then organised in schemas.

But much of this process–object theory does not resonate with a great deal of what I see in my mathematics classroom. For example, it does not help me explain or describe what is happening when a learner fumbles around with ‘new’ mathematical signs making what appear to be arbitrary connections between these new signs and other apparently unrelated signs. Similarly, it does not explain how these incoherent–seeming activities can lead to usages of mathematical signs that are both acceptable to professional members of the mathematical world and that are personally meaningful to the learner.

I suggest that the central drawback of these neo–Piagetian theories is that they are rooted in a framework in which conceptual understanding is regarded as deriving largely from interiorised actions; the crucial role of language (or signs) and the role of social regulation and the social constitution of the body of mathematical knowledge is not integrated into the theoretical framework.

What is required is a framework in which the link between an individual’s construction of a concept and social knowledge (existing in the community of mathematicians and in reified form in textbooks) is foregrounded. Furthermore, given that mathematics can be regarded as the “quintessential study of abstract sign systems” (Ernest, 1997) and mathematics education as “the study of how persons come to master and use these systems” (ibid.), a framework which postulates semiotic mediation as the mechanism of learning, seems apposite. I claim that Vygotsky’s much–neglected theory of concept formation, allied with his notion of the functional usage of a sign (1986), is such a framework.

## **VYGOTSKY’S THEORY OF CONCEPT FORMATION**

Although Vygotskian theory (but not the theory of concept formation) has been applied extensively in mathematics education, most of the research has focused on the mathematical activities of a group of learners or a dyad rather than the individual (Van der Veer and Valsiner, 1994). Furthermore it has been applied most frequently to primary school or high school learners (for example, van Oers, 1996; Radford, 2001) rather than to individuals at undergraduate level.

Indeed, Van der Veer and Valsiner (1994) claim that the use of Vygotsky in the West has been highly selective. In particular they argue that “*the focus on the individual developing person* which Vygotsky clearly had ... has been persistently overlooked” (p. 6; italics in original).

It is important to note that a focus on the individual (possibly with a textbook or in consultation with a lecturer) does not contradict the fundamental Vygotskian notion that “social relations or relations among people genetically underlie all higher functions and their relationships” (Vygotsky, 1981, p. 163). After all, a situation consisting of a learner with a text is necessarily social; the textbook or exercises have been written by an expert (and can be regarded as a reification of the expert’s ideas); also the text may have been prescribed by the lecturer with pedagogic intent. Thus a focus on the individual does not undermine the significance of the social.

### **Functional use of the sign**

In order to understand Vygotsky’s theory, one needs to understand how Vygotsky used the term ‘word’. Vygotsky regarded a word as embodying a generalisation and hence a concept.

As such, Vygotsky postulated that the child uses a word for communication purposes before that child has a fully developed understanding of that word. As a result of this use in communication, the meaning of that word (i.e., the concept) evolves for the child:

Words take over the function of concepts and may serve as means of communication long before they reach the level of concepts characteristic of fully developed thought (Uznadze, cited in Vygotsky, 1986: 101).

The use of a word or sign to refer to an object (real or virtual) prior to ‘full’ understanding resonates with my sense of how an undergraduate student makes a new mathematical object meaningful to herself. In practice, the student starts communicating with peers, with lecturers or the potential other (when writing) using the signs of the new mathematical object (symbols and words) before she has full comprehension of the mathematical sign. It is this communication with signs that gives initial access to the new object.

It is a functional use of the word, or any other sign, as a means of focusing one’s attention, selecting distinctive features and analysing and synthesizing them, that plays a central role in concept formation (Vygotsky, 1986: 106).

Secondly but closely linked to the above notion, is Vygotsky’s argument that the child does not spontaneously develop concepts independent of their meaning in the social world:

He does not choose the meaning of his words... The meaning of the words is given to him in his conversations with adults (Vygotsky, 1986: 122).

That is, the meaning of a concept (as expressed by words or a mathematical sign) is ‘imposed’ upon the child and this meaning is not assimilated in a ready-made form. Rather it undergoes substantial development for the child as she uses the word or sign in her communication with more socialised others.

Thus the social world, with its already established definitions (as given in dictionaries or books) of different words, determines the way in which the child's generalisations need to develop.

Analogously, I argue that in mathematics, a student is expected to construct a concept whose use and meaning is compatible with its use in the mathematics community. To do this, that student needs to use the mathematical signs in communication with more socialised others (including the use of textbooks which embody the knowledge of more learned others). In this way, concept construction becomes socially regulated.

### **Semiotic mediation**

Vygotsky (1978) regarded all higher human mental functions as products of mediated activity. The role of the mediator is played by a psychological tool or sign, such as words, graphs, algebra symbols, or a physical tool. These forms of mediation, which are themselves products of the socio-historical context, do not just facilitate activity; they define and shape inner processes. Thus Vygotsky saw action mediated by signs as the fundamental mechanism which links the external social world to internal human mental processes and he argued that it is

by mastering semiotically mediated processes and categories in social interaction that human consciousness is formed in the individual (Wertsch and Stone, 1985: 166).

Allied to this, concept formation, as discussed above, is only possible because the word or mathematical object can be expressed and communicated via a word or sign whose meaning is already established in the social world.

In mathematics, the same mathematical signs mediate two processes: the development of a mathematical concept in the individual and that individual's interaction with the already codified and socially sanctioned mathematical world (Radford, 2000). In this way, the individual's mathematical knowledge is both cognitively and socially constituted.

This dual role of a mathematical sign by a learner before 'full' understanding is not well appreciated by the mathematics education community; indeed, its manifestations in the form of activities such as manipulations, imitations and associations are often regarded disparagingly by mathematics educators. That is, they regard such activities as 'meaningless' and without worth. (Conversely, back-to-basics mathematics educators may regard adequate use of a mathematical sign as sufficient evidence of a student's understanding of the relevant mathematical concept. Of course, in terms of Vygotsky's theory, this is not the case).

Vygotsky's theory, that usages of the sign are a necessary part of concept formation, manages to provide a link between certain types of mathematical activities (including those activities regarded pejoratively by many educators) and the formation of concepts.

## Different stages

Vygotsky further elaborated his theory by detailing the stages in the formation of a concept. He claimed that the formation of a concept entails different preconceptual stages (heaps, complexes and potential concepts).

During the syncretic heap stage, the child groups together objects or ideas which are objectively unrelated. This grouping takes place according to chance, circumstance or subjective impressions in the child's mind. In the mathematical domain, a student is using heap thinking if she associates one mathematical sign with another because of, say, the layout of the page.

The syncretic heap stage gives way to the complex stage. In this stage, ideas are linked in the child's mind by associations or common attributes which exist objectively between the ideas.

Complex thinking is crucial to the formation of concepts in that it allows the learner to think in coherent terms and to communicate via words and symbols about a mental entity. And, as I have argued above, it is this communication with more knowledgeable others which enables the development of a personally meaningful concept whose use is congruent with its use by the wider mathematical community.

Complexes corresponding to word meanings are not spontaneously developed by the child: The lines along which a complex develops are predetermined by the meaning a given word already has in the language of adults (Vygotsky, 1986: 120).

Furthermore, in complex thinking the learner begins to abstract or isolate different attributes of the ideas or objects, and the learner starts organizing ideas with particular properties into groups thus creating the basis for later more sophisticated generalisations.

With complex thinking, the learner is not using logic; rather she is using some form of non-logical or experiential association. Thus complex thinking often manifests as bizarre or idiosyncratic usage of mathematical signs.

For example, the learner is using complex thinking when she associates the properties of a 'new' mathematical sign with an 'old' mathematical sign with which she is familiar and which is epistemologically more accessible.

As an illustration, on first encountering the derivative,  $f'(x)$ , of a function  $f(x)$ , the learner may associate the properties of  $f'(x)$  with the properties of  $f(x)$ . Accordingly, many learners assume or imply that since  $f(x)$  is continuous, so is  $f'(x)$ . Clearly this is not logical; indeed it is mathematically incorrect.

Another example of activity guided by complex thinking is when the student seems to focus on a particular aspect of the mathematical expression and to associate these symbols or words with a new sign. For instance, when dealing with the greatest integer function  $\lceil x \rceil = \text{greatest integer } \leq x$ , many students latch onto the word 'greatest' ignoring the condition  $\leq x$ . They then link the word 'greatest' to the idea of

‘greater than’ and accordingly state that, say,  $\lceil \lceil 4.3 \rceil \rceil = 5$  (whereas of course, the answer should be 4).

My point here is not *how* the student uses the signs but rather *that* she uses the signs. Through this use, the student gains access to the ‘new’ mathematical object and is able to communicate (to better or worse effect) about it. Through social regulation or reflection (in tandem with the socially constituted definition and for an attenuated or extended time period) the learner will eventually come to use and understand the signs in ways that are congruent with official mathematics.

My observations of undergraduate students over the years ties in very well with the idea that preconceptual thinking is a necessary part of successful mathematics concept construction (this is evidenced by many of these students’ apparently confused mathematical assertions prior to mathematical coherence). Of course, the time spent using complex thinking may be very brief or very long, depending on the student, the particular mathematical object, the task, the context and the social interventions.

Vygotsky distinguished between five different types of complexes. For the purposes of this talk it is sufficient to elaborate on the pseudoconcept, which is a construct which effectively bridges the divide between the individual and the social and between complex and concept. (For elaboration and exemplification of the different types of complexes, see Sierpinska (1993), Berger (2004a, 2004b)).

### **The pseudoconcept: a bridge between the individual and the social**

In order to understand the pseudoconcept one needs to know how Vygotsky used the word ‘concept’: in a concept, the bonds between the parts of an idea and between different ideas are logical and the ideas form part of a socially-accepted system of hierarchical knowledge.

According to Vygotsky, the transition from complexes to concepts is made possible by the use of pseudoconcepts. Hence the pseudoconcept is a very special form of complex.

Pseudoconcepts resemble true concepts in their use, but the thinking behind these pseudoconcepts is still complex in character. This is because the bonds between the different elements of a pseudoconcept are associative and experiential rather than logical and abstract. But the learner is able to use the pseudoconcept in communication and activities *as if* it were a true concept.

The use of pseudoconcepts is ubiquitous in mathematics and is analogous to a child using a word in conversation with an adult before fully understanding the meaning of that word. Pseudoconcepts occur whenever a student uses a particular mathematical object in a way that coincides with the use of a genuine concept, even though the student has not fully constructed that concept for herself. For example, a student may use the definition of the derivative of a function to compute the derivative of the function before she ‘understands’ the nature of the derivative or its properties.

Vygotsky (1986) argued that the use of pseudoconcepts enables children to communicate effectively with adults and that this communication (the intermental aspect) is necessary for the transformation of the complex into a genuine concept (the intramental aspect) for the learner.

Verbal communication with adults (...) become a powerful factor in the development of the child's concepts. The transition from thinking in complexes to thinking in concepts passes unnoticed by the child because his pseudoconcepts already coincide in content with adult concepts (Vygotsky, 1986: 123).

Thus the pseudoconcept functions as the bridge between concepts whose meaning is more or less fixed and constant in the social world (such as that body of knowledge we call mathematics) and the learner's need to make and shape these concepts so that they become personally meaningful. This bridging function of the pseudoconcept is the basis for my contention that the pseudoconcept can be regarded as the link between the individual and the social. As such pseudoconcepts are a necessary stage in the child's or student's development of true concepts. Furthermore the notion of the pseudoconcept is entirely consistent with the functional use of a sign.

The pseudoconcept can be used to explain how the student is able to use mathematical signs (in algorithms, definitions, theorems, problem-solving, and so on) in effective ways that are commensurate with that of the mathematical community even though the student may not fully 'understand' the mathematical object. The hope is that through appropriate use and social interventions, the pseudoconcept will get transformed into a concept.

## CONCLUSION

In this paper, I have argued that Vygotsky's theory of concept formation provides an apposite framework within which to elaborate how an individual constructs a concept that is personally meaningful and whose usage is commensurate with that of the mathematical community.

In particular, I argued that the notion of functional usage of the sign, together with the construct of the pseudoconcept, can be used to bridge the divide between an individual's concept formation and a socially sanctioned mathematical definition. Related to this, idiosyncratic mathematical activities can be regarded as manifestations of complex thinking. With social regulation, these complexes can be transformed into pseudoconcepts and ultimately concepts can be formed. Finally, I argued that Vygotsky's notion that all knowledge is semiotically mediated is necessary for understanding how students use mathematical signs to gain access to mathematical objects.

What is now required is empirical research which illuminates the bridges between personal and socially sanctified usages of mathematical signs, explicates the transformations from complexes to pseudoconcepts to concepts, and explores the relationships between different usages of signs and meaning-making.

## References

- Berger, M. (2004a). Heaps, complexes and concepts (part 1), *For the Learning of Mathematics* 24 (2), pp. 2 – 6.
- Berger, M. (2004b). Heaps, complexes and concepts (part 2), *For the Learning of Mathematics* 24 (3).11 – 17.
- Czarnocha, B., Dubinsky, E., Prabhu, V. & Vidakovic, D. (1999). One theoretical perspective in undergraduate mathematics education research. In O. Zaslavsky (Ed.), *Proc. 23<sup>rd</sup> Conf. of the Int. Group for the Psychology of Mathematics Education*, (Vol. 1, pp. 95–110). Haifa, Israel: PME
- Dubinsky, E. (1991). Reflective abstraction in advanced mathematical thinking. In D. Tall (Ed.) *Advanced Mathematical Thinking* (pp. 95–123). Dordrecht: Kluwer Academic.
- Ernest, P. (1997). Introduction: Semiotics, Mathematics and Mathematics Education, *Philosophy of Mathematics Education*, Journal 10.  
<http://www.ex.ac.uk/~PERnest/pome10/art1.htm>
- Radford, L. (2000). Signs and meanings in students' emergent algebraic thinking: A semiotic analysis, *Educational Studies in Mathematics* 42: 237–268.
- Radford, L. (2001). Factual, Contextual and symbolic generalizations in algebra. In M. van den Heuvel–Panhuizen (Ed.) *Proc. 25<sup>th</sup> Conf. of the Int. Group for the Psychology of Mathematics Education*, (Vol. 4, pp. 81–88). Utrecht, The Netherlands: PME.
- Sierpinska, A. (1993). The development of concepts according to Vygotsky, *Focus on Learning Problems in Mathematics*, Vol. 15, Nos. 2 & 3, 87–107.
- Tall, D.O. (1995). *Cognitive Growth in Elementary and Advanced Mathematical Thinking*, In L. Meira and D. Carraher. (Eds.) *Proc. 19<sup>th</sup> Conf. of the Int. Group for the Psychology of Mathematics Education*, (Vol. 1, pp. 61–75). Recife, Brazil: PME.
- Tall, D.O., Thomas, M., Davis, G., Gray, E., & Simpson, A. (2000). What is the object of the encapsulation of a process? *Journal of Mathematical Behavior*, 18, 2, 1–19.
- Van der Veer & Valsiner (1994). Reading Vygotsky: from fascination to construction. In R. van der Veer & J Valsiner (Eds. and Trans.), *The Vygotsky Reader* (pp. 1–9). Oxford: Blackwell Publishers.
- Van Oers, B. (1996). Learning mathematics as a meaningful activity. In P. Nesher, L.P. Steffe, P. Cobb, G. Goldin, & B. Greer (Eds.) *Theories of mathematical learning* (pp. 91–113). Mahwah, N.J.: Lawrence Erlbaum.
- Vygotsky, L.S. (1981). The genesis of higher mental functions. In J.V. Wertsch (ed.) *The Concept of Activity in Soviet Psychology* (pp.144–188). Armonk, NY: M.E. Sharpe.
- Vygotsky, L. S. (1986). *Thought and Language*, A. Kozulin, (Ed. and Trans.), Cambridge, MA.: MIT Press.
- Wertsch, J.V. & Stone, C.A. (1985). The concept of internalisation in Vygotsky's account of the genesis of higher mental functions. In Wertsch, J.V. (Ed.) *Culture, Communication and Cognition* (pp. 162–179). New York: Cambridge University Press.