A WHOLE-SCHOOL APPROACH TO DEVELOPING MENTAL COMPUTATION STRATEGIES

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Curriculum documents in Australia and elsewhere emphasize the importance of mental computation. There has been, however, little advice about developing and implementing mathematics programs that have mental computation strategies as a focus. One primary school's response to this issue was to identify generic mental computation strategies that could be developed across the school in all grades from Kindergarten to Grade 6. The program was implemented within a framework for quality teaching. Initial results suggest that teachers have developed flexible approaches to teaching mental computation. Students are more aware and articulate about the nature of the strategies that they use, and students' learning outcomes have improved.

INTRODUCTION

There is a growing emphasis on the place of mental computation in schools. In Victoria, Australia, for example, the curriculum framework places sufficient weight on mental computation that it is a separate sub-strand. This sub-strand describes expected outcomes in terms of a progression from whole numbers and recall of basic facts, through recognition of decimal and fraction equivalences to the use of a range of strategies to compute mentally with fractions, decimals and percents (Victorian Curriculum and Assessment Authority, 2002). In the UK, mental computation is highlighted in the National Numeracy Strategy as a deliberate process that involves students in developing efficient and effective approaches to calculation (Askew, 2003).

The development of number understanding in the early years is well documented (e.g., Wright & Gould, 2002) and there is continuing research into children's understanding of written computation (e.g., Anghileri, 2004). It is well known that students draw on a range of formal and informal strategies when computing mentally, and strategy development is advocated as an effective approach (McIntosh, 2003).

Jacaranda Public School[1] in central New South Wales (NSW), Australia, took an unusual approach to changing the emphasis of its mathematics programs away from drill and practice of written algorithms to developing mental computation strategies, in the context of applying a Quality Teaching Model (NSW Department of Education and Training (DET), 2003). The program that the staff developed had two major aims: to change pedagogy as a consequence of implementing the Quality Teaching

[1] Names have been changed to preserve confidentiality.

2005. In Chick, H. L. & Vincent, J. L. (Eds.). *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 2, pp. 201-208. Melbourne: PME.

Model, and to transform the mathematics curriculum through an emphasis on mental computation. This paper reports the initial evaluation findings.

The NSW Quality Teaching Model

The model of quality teaching adopted in NSW has three dimensions: Intellectual Quality, Quality Learning Environment, and Significance (DET, 2003). Each dimension is composed of six elements that are used to identify quality teaching in classroom situations. The dimensions and elements are summarised in Table 1.

Intellectual Quality	Quality Learning Environment	Significance
Deep knowledge	Explicit quality criteria	Background knowledge
Deep understanding	Engagement	Cultural knowledge
Problematic knowledge	High expectations	Knowledge integration
Higher-order thinking	Social support	Inclusivity
Metalanguage	Students' self-regulation	Connectedness
Substantive communication	Student direction	Narrative

Table 1: Dimensions and elements of the NSW model of quality teaching

To make the model operational for teachers, each of the elements was first considered from the perspective of mental computation. The promotion of Deep Knowledge, for example, implied a focus on strategy development, linking to key concepts such as place value; Social Support in the classroom indicated that all students would be encouraged to share contributions and accept different approaches to computation; Cultural Knowledge recognized that different groups in the community had various ways of undertaking mental computations and that these would be explicitly discussed. Every element was interpreted in this manner.

Having established how the model could be applied to mental computation, the school staff decided that the program would be based on a whole-school approach. For two weeks at a time, the whole school, from Kindergarten to Year 6, would focus on applying a particular mental computation strategy to content appropriate to the grade level and experience of the students, copying a process that the school had used successfully for writing development. This step proved quite challenging. Although teachers could identify strategies, understanding what this meant across the full range of grades was not simple, and there was considerable discussion at staff meetings about the mathematics program and aspects that were the keys to students' developing skills and understanding. Ten target ideas were identified for the program's focus, including processes for calculation, such as visualising and counting on, and basic concepts, such as place value and pattern recognition. 'Games with a point' was also included to encourage a move away from reliance on text books. Each of the target strategies was then described for every grade with specific links to the NSW syllabus (NSW Board of Studies, 2002). The description of the 'Using Patterns' strategy for each grade is shown in Table 2.

<table>
<tr><th>Grade</th><th>Scope</th><th>Examples</th><th>Outcomes*</th></tr>
<tr><td>K</td><td>Copying, and continuing simple patterns</td><td>Continues the pattern ● ■ ♦</td><td>PAES1.1</td></tr>
<tr><td>1</td><td>Finding missing elements in a pattern</td><td>Completes 4, 7, _, 13, _, 19</td><td>PAS1.1</td></tr>
<tr><td>2</td><td>Systematically uses number combinations</td><td>Writes all combinations of 6: 6 + 0, 5 + 1, 4 + 2, 3 + 3, 2 + 4, 1 + 5, 0 + 6</td><td>PAS1.1</td></tr>
<tr><td>3</td><td>Completes number patterns based on tables</td><td>Completes 88, 80, _, 64, 56, _, 40</td><td>PAS2.1</td></tr>
<tr><td>4</td><td>Systematically uses number combinations based on multiplication and division</td><td>Writes all combinations of 24:
6 x 4, 4 x 6, 24 ÷ 6 = 4, 24 ÷ 4 = 6</td><td>PAS2.1</td></tr>
<tr><td>5</td><td>Uses and describes number patterns based on one operation in different ways</td><td>Completes a table of values e.g.,<table><tr><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>3</td><td>6</td><td></td><td>12</td></tr></table>and describes the pattern in words, "It's the three times tables", or as a rule: "To get the bottom number, times the top number by 3".</td><td>PAS3.1a</td></tr>
<tr><td>6</td><td>Uses patterns with fractions and decimals to make calculations easier.</td><td>7 x 11 = 77 so 7 x 1.1 = 7.7
1/3 is the same as 2/6, so
1/3 + 1/6 = 3/6 which is ½.</td><td>PAS3.1b</td></tr>
</table>

Table 2: Description of the 'Using Patterns' strategy across grades

* The outcomes refer to the NSW K-6 Mathematics Syllabus (NSW Board of Studies, 2002).

The activities involved were not confined solely to mental computation. There was recognition that recording was an important part of mathematical activity, but the focus of that recording was shifted away from formal written algorithms to purposeful recording of students' thinking.

PROGRAM EVALUATION

Methodology

During the period from July to December 2004, the program was formally evaluated.

Evaluation was limited to the primary years, Grades 3 to 6, at the request of the school's teachers who wanted to continue to use existing processes in the early years.

A sample of approximately 90 students from four classes, covering Grades 3 to 6, undertook tests of mental computation in early August and late November, using the tests and methodology developed for an earlier study (Callingham & McIntosh, 2001). Students in Grades 3 and 4 took a test of 50 items, and students in Grades 5 and 6 took a longer version of 65 items. The researcher administered all tests. Questions were presented orally using a CD, and students wrote their answers on provided response sheets. Different test forms were used for the pre- and post-tests, but all tests had overlapping items so that they could be linked using Rasch measurement techniques. The responses were entered verbatim into a spreadsheet to provide for further error analysis, and then scored as correct/incorrect. The scored responses were scaled using Rasch measurement techniques using Quest computer software (Adams & Khoo, 1996), anchored to baseline values obtained from the earlier study so that they could be directly compared. Students' performances were estimated in logits (the natural logarithm of the odds of success), the unit of measure used for Rasch measurement. Using a pre- and post-test model provided performance measures at two points in time, and a growth measure over the 15 week period between the tests.

Additional information was collected from lesson observations in each class. Each lesson lasted approximately 40 minutes, and was part of the regular program, not specially prepared. The researcher observed the lesson informally, interacting and talking with the students, making brief notes. Immediately after the lesson ended the notes were written into a coherent account of the lesson, and this was discussed with the teacher concerned. The discussion confirmed the focus of the lesson and allowed teachers and the researcher to agree about the lesson description. Lesson observations were analyzed using the framework provided by the NSW Quality Teaching Model (DET, 2003).

Interviews were conducted with 12 students, three from each grade, chosen by their teachers to cover the range of competence in their class, or because the students had unusual strategies. The students were interviewed twice, first in a group to establish a relationship with the researcher, and then individually. The interview protocol was adapted from one used elsewhere (Caney, 2002) and focused on mental computation strategies used by students.

Initial Results

The results reported include initial analysis of students' performance and growth, lesson observations and interviews, and do not include error analysis. Pre- and post-test results were available for 89 students (18 in Grade 3; 22 in Grade 4; 28 in Grade 5; 21 in Grade 6), although the actual sample was slightly larger. Table 3 shows the mean scores in logits from the two test administrations. The school's mean score improved overall by 0.66 logits. A paired sample t-test indicated that the change was highly significant (t = -4.10, p = 0.000).

	N	Minimum	Maximum	Mean	Std. Deviation
August	95	-4.41	4.19	0.43	2.15
November	99	-3.69	5.67	1.09	2.12

Table 3: Mean performance overall.

Improvement was more marked in the lower grades than in Grades 5 and 6, as shown by the boxplots in Figure 1, where the median and 25th percentile values in Grades 3 and 4 shifted upwards markedly. In Grades 5 and 6 improved performance appeared to be mainly among the upper 25 percent, as shown by the extended 'whisker' at the top of the boxes. Grade 5 also showed an extended lower whisker in the post-test, suggesting that there was considerable variation in performance in this grade.

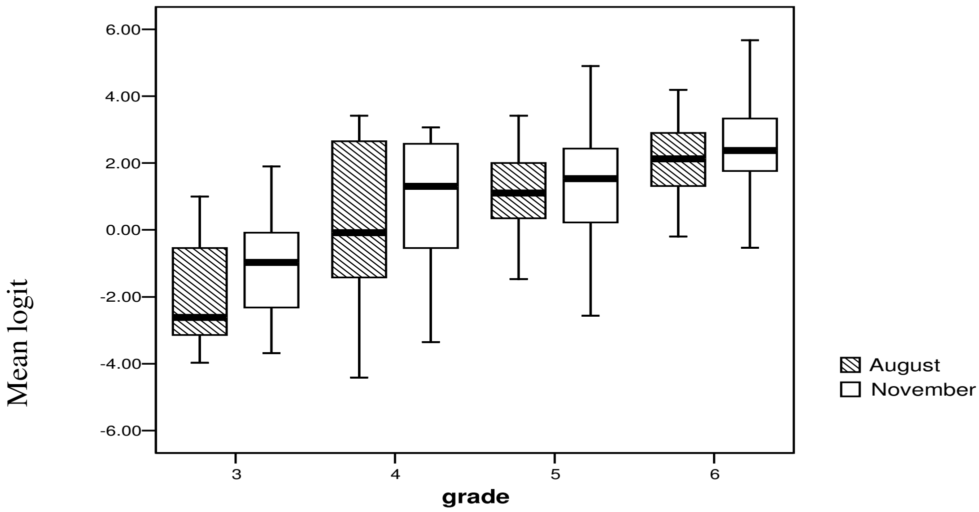

Figure 1: Mean performance by grade.

The four lesson observations were analyzed using the framework provided by the dimensions and elements of the NSW Quality Teaching Model (DET, 2003). All observed lessons were characterized by a very clear and explicit focus – the Deep Knowledge element of the model. Although there were several activities undertaken in each lesson, each addressed the target idea in a different way. For example, a Grade 5 lesson that focused on the doubling strategy included a game in which students had to either double or half the given number and get four correct answers in a row on a grid that they created for themselves. The lesson continued by using the arrangement of desks (4 x 4 grid) as a stimulus to discuss different ways in which number sentences could be made. Finally students were given a number and asked to create two number sentences to make that number. These were shared and discussed, with particular reference to the doubling strategy.

There was considerable discussion and interaction between teacher and students throughout all lessons. Students clearly articulated their strategies, demonstrating the

Deep Understanding and Metalanguage elements of the model. All students were confident about taking risks and suggesting alternative approaches, suggesting that the Quality Learning Environment dimension was well established. In every lesson students were on task throughout the lesson and there was little wasted time, demonstrating the Engagement element. This was probably encouraged by the variety of activities and 'pace' of each lesson observed. Although each activity was completed, in a typical 40-minute lesson there were three or four short activities, each addressing the target idea in a different way.

Teachers also made clear and explicit links to prior experiences of the class. A lesson in a Grade 3 and 4 composite class, for example, started with a discussion about patterns in the nine times table that the class had worked on in the previous week. The rest of the lesson took place in the computer room, where the students practised their previously learnt skills of making a table by creating a grid in which they had to provide number sentences to make given nine times table answers in ten different ways. This teacher skillfully linked the lesson focus of working backwards to prior knowledge, pattern recognition and other learning areas, showing several of the elements of the Significance dimension of the model. This lesson was particularly effective in allowing all students to participate at their own level. Some students took a random approach to finding number sentences, others realised that they could use inverse operations such as making 18 by multiplying it by various numbers and then dividing the answer by the multiplicand. In this way they were able to generate the ten required number sentences quickly.

Teachers were addressing many aspects of the Quality Teaching Model, although most indicated that they did not consciously do this in their planning. It seems likely that the model captures many of the features that contribute to good teaching, and these very competent teachers were drawing on the elements unconsciously. All teachers, however, did indicate that they had changed their teaching to address mental computation more explicitly, with less emphasis on drill and practice of written algorithms. They suggested that their lessons involved students more in discussion, and that they developed a clear focus for every lesson, depending on the target strategy.

Anecdotal evidence from discussions with the school's principal indicated that this changed approach was being implemented across the school. Teachers who had been reluctant to change their mathematics approach, were reporting that their students were more able to talk about their mathematics, and used a wider range of strategies than had been recognized before the program had started.

Students' understanding of strategies and flexibility in strategy use was confirmed by the interviews. In most instances, students were aware of their strategies and could explain them in some detail, even students who were relatively less skilled. Some students were very good at making groups of 10, and could use these ideas flexibly to solve extension problems. Weaker students tended to use counting on strategies, sometimes inappropriately, such as counting on by ones from 24 when asked "How would you work out 24 add 8?" Stronger students would say something like "Take 6 from 8 and add it to 24 to make 30 and then add 2".

Some of the younger students, not unexpectedly, tended to use repeated addition for some of the multiplication problems, for example counting by 3 seven times to calculate 7 x 3, or, for more complex problems such as 24 x 3, counting on by three 24 times. Although inefficient, these students did have a strategy that would lead to a correct solution and, more importantly, were prepared to try these out. There were very few instances of students not being willing and able to attempt a problem, even when the problem was difficult.

Many students showed high levels of "number sense", intuitively recognizing an incorrect answer and correcting this. Student Z (Grade 5), for example, in response to 0.5 + 0.5 answered correctly, and then explained "I originally thought zero point ten but it's lower than 0.5". His explanation was confused, but he clearly had a feeling for the size of the numbers involved. Some very clearly described visualisation strategies, including "… seeing fingers in my head and counting them" (Student J, Grade 4). In general, younger students seemed more flexible in their thinking. Some of the older students made extensive, and accurate, use of a written strategy, with one girl physically writing the problems on the desk with a finger.

The interviews supported the lesson observations in that students could clearly articulate their strategies, and were confident about using them. Some students had idiosyncratic but effective approaches that they drew on as appropriate.

DISCUSSION

The changed whole school approach to mathematics teaching appeared to have led to improved outcomes for students on standardized tests of Mental Computation Competence. Although some of this improvement may well have been due to increased familiarity with the test format, students also demonstrated clear understanding of many mental computation strategies that they could use effectively, both in a classroom and an interview setting. Teachers reported that their teaching had shifted its focus from written algorithms to strategy and concept development, and this was borne out by the lessons observed. The lessons also addressed many aspects of the NSW Quality Teaching Model adopted by the school. The combined effect of improving teaching quality and a focus on mental computation strategies appears to have been effective.

The whole school focus, across Grades K to 6, on a particular mental computation strategy is unusual. Despite these promising initial results, before any recommendations could be made regarding this approach further work is needed about its efficacy. For example some strategies may be more effective with particular content, such as whole numbers, or strategies may be more appropriately developed in particular grades rather than used across the school. There is also a need to consider further the differential effects shown across the grades, and to establish whether the lower growth observed in the upper primary grades is affected by the strategies used. It may also be possible to establish a hierarchy of strategies that is developmental in nature. These initial results, however, appear to have potential to inform program development in mental computation.

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