

MATHEMATICAL MODELLING WITH 9-YEAR-OLDS

Lyn D. English and James J. Watters

Queensland University of Technology

This paper reports on the mathematical modelling of four classes of 4th-grade children as they worked on a modelling problem involving the selection of an Australian swimming team for the 2004 Olympics. The problem was implemented during the second year of the children's participation in a 3-year longitudinal program of modelling experiences (i.e., grades 3-5; 2003-2005). During this second year the children completed one preparatory activity and three comprehensive modelling problems. Throughout the two years, regular teacher meetings, workshops, and reflective analysis sessions were conducted. The children displayed several modelling cycles as they worked the Olympics problem and adopted different approaches to model construction. The children's models revealed informal understandings of variation, aggregation and ranking of scores, inverse proportion, and weighting of variables.

INTRODUCTION

With the increased importance of mathematics in our ever-changing global market, there are greater demands for workers who possess more flexible, creative, and future-oriented mathematical and technological capabilities. Powerful mathematical processes such as constructing, describing, explaining, predicting, and representing, together with quantifying, coordinating, and organising data, provide a foundation for the development of these capabilities. Also of increasing importance is the ability to work collaboratively on multi-dimensional projects, in which planning, monitoring, and communicating results are essential to success (Lesh & Doerr, 2003).

Several education systems are thus beginning to rethink the nature of the mathematical experiences they should provide their students, in terms of the scope of the content covered, the approaches to student learning, ways of assessing student learning, and ways of increasing students' access to quality learning. One approach to addressing these concerns is through mathematical modelling (English & Watters, 2004). Indeed, a notable finding across studies of professionals who make heavy use of mathematics is that a facility with mathematical modelling is one of the most consistently needed skills (Gainsburg, 2003; Lesh & Zawojewski, in press).

Traditionally, students are not introduced to mathematical modelling until the secondary school years (e.g., Stillman, 1998). However, the rudiments of mathematical modelling can and should begin much earlier than this, when young children already have the foundational competencies on which modelling can be developed (Diezmann, Watters, & English, 2002; Lehrer & Schauble, 2003). This paper addresses the mathematical modelling processes of children from four classes of nine-year-olds (4th-grade), who are participating in a three-year longitudinal

program of modelling experiences. The children commenced the program in their third-grade, where they completed preparatory modelling activities prior to working comprehensive modelling problems (English & Watters, in press).

MATHEMATICAL MODELLING FOR THE PRIMARY SCHOOL

The problem-solving experiences that children typically meet in schools are no longer adequate for today's world. Mathematical problem solving involves more than working out how to go from a given situation to an end situation where the "givens," the goal, and the "legal solution steps" are specified clearly. The most challenging aspect of problems encountered in many professions today involve developing useful ways of thinking mathematically about relevant relationships, patterns, and regularities (Lesh & Zawojewski, in press). In other words, problem solvers need to develop more productive ways of interpreting and thinking about a given problematic situation. Interpreting a situation mathematically involves modelling, where the focus is on the structural characteristics of the situation, rather than the surface features (e.g. biological, physical or artistic attributes; English & Lesh, 2003).

The modelling problems of the present study require children to generate mathematical ways of thinking about a new, meaningful situation for a particular purpose (e.g., to determine which set of conditions is more suitable for growing certain types of beans; English & Watters, 2004). In contrast to typical school problems, modelling tasks do not present the key mathematical ideas "up front." Rather, the important mathematical constructs are embedded within the problem context and are elicited by the children as they work the modelling problem. The problems allow for multiple approaches to solution and can be solved at different levels of sophistication, thus enabling all children to have access to the important mathematical content.

The problems are multifaceted in their presentation and include background information on the problem context, "readiness questions" on this information, detailed problem goals, tables of data, and supporting illustrations. In turn, the problems call for multifaceted products (models). The nature of these products is such that they reveal as much as possible about children's ways of thinking in creating them. Importantly, the models that children create should be applicable to other related problem situations; to this end, we have presented children with sets of related problems that facilitate model application (English & Watters, in press).

The problems require the children to explain and justify their models, and present group reports to their class members. Because their models are to be sharable and applicable to classes of related situations, children have to ensure that what they produce is informative, "user-friendly," and clearly and convincingly conveys the intended ideas and ways of operating with these. Because the problems are designed for small group work, each child has a shared responsibility to ensure that their product does meet these criteria.

RESEARCH DESIGN AND APPROACH

Multilevel collaboration, which employs the structure of the multitiered teaching experiments of Lesh and Kelly (2000) and incorporates Simon's (2000) case study approach to teacher development, is being employed in this study. Such collaboration focuses on the developing knowledge of participants at different levels of learning. At the first level, children work on sets of modelling activities where they construct, refine, and apply mathematical models. At the next level, classroom teachers work collaboratively with the researchers in preparing and implementing the child activities. At the final level, the researchers observe, interpret, and document the knowledge development of all participants (English, 2003). Multilevel collaboration is most suitable for this study, as it caters for complex learning environments undergoing change, where the processes of development and the interactions among participants are of primary interest (Salomon, Perkins, & Globerson, 1991).

Participants

Four 4th-grade classes (9 years) participated in the second year of this study, after having also participated in the first year. One of the four class teachers had also been involved in the first year of the study, whereas the remaining three teachers were new to the study. The classes represented the entire cohort of fourth graders in a school situated in a middle-class suburb of Brisbane, Australia. The school principal and assistant principal provided strong support for the project's implementation.

Procedures and activities

At the beginning of the year, a half-day professional development workshop was held with the teachers where we outlined the project and negotiated plans for the year. The four teachers involved in the first year of the study also provided input by sharing their experiences and highlighting what they had learned about implementing modelling activities, as well as describing student learning that had occurred.

An initial preparatory activity (focusing on reading and interpreting data) and three modelling problems were implemented during the year. The first modelling activity was conducted in winter over four weeks and focused on "Skiing for the First Time." The second problem focused on the "Olympics," which was pending at the time of the activity, and the third was conducted during a theme on weather and required the children to decide where to locate a resort in a region subject to Cyclones.

The Olympics problem was undertaken with children working in groups of three or four in four 40-minute lessons conducted over two weeks. Audio-taped meetings were held with teachers to plan the lessons beforehand and to analyse outcomes immediately on conclusion of the activity. The children were presented with an initial readiness activity containing background information on the history of the Olympics and a table of data displaying the men's world 100 metre freestyle records from 1956 to 2000. The children were to answer a number of questions about the information and data. They were then presented the main modelling problem comprising (a) the

data displayed in Table 1; (b) the accompanying information: *We (Australia's Olympic Swimming Committee) need to make sure that we have selected our best swimmers. The Olympic Swimming Committee has already selected the women's swim team. However, they are having difficulty in selecting the most suited swimmers for competing in the men's 100m freestyle. The Olympic Swimming Committee has collected data on the top seven (7) male swimmers for the 100m freestyle event. The data collected (see Table 2; Table 1 in this paper) show each of the swimmer's times over the last ten (10) competitions. It has been decided by the Olympic Swimming Committee to have you as part of their selection team;* and (c) the problem goal: *Being selectors for the Olympic Swimming Committee, you need to use the data in Table 2 to develop a method for selecting the two (2) most suited swimmers for the Men's 100m Freestyle event. Write a report to the Olympic Swimming Committee telling them who you selected and why. You need to also explain the method you used in selecting these swimmers. The selectors will then be able to use your method in selecting the most suited swimmers for the other swimming events.*

Data Collection and Analysis

Each of the four teachers was fitted with a radio microphone and videotaped during the lesson so that her dialogue with children could be monitored. A second video camera captured critical events as they occurred or was focused on selected groups of students to monitor student interactions. Audio recordings of conversations among children and with teachers complemented video data. Other data sources included classroom field notes, children's artefacts (including their written and oral reports), and the children's responses to their peers' feedback in the oral reports. In our data analysis, we employed ethnomethodological interpretative practices to describe, analyse, and interpret events (Erickson, 1998).

FINDINGS

From our analysis of the children's transcripts as they worked the modelling problems and reported to their peers, we identified a number of different approaches to model development. These included: (a) focusing on personal best times (PBs) only, with some groups also considering the extent of a swimmer's variation from his PB; (b) tallying the number of winning races for each swimmer in each event, and comparing the totals; (c) aggregating the two or three lowest times of each swimmer and comparing the totals; (d) assigning scores (and weighted scores) to the two lowest times of each swimmer and aggregating the scores; (e) in addition to [d], assigning weighted scores to the two lowest PBs, aggregating all the scores, and then ranking the totals (refer Figure 1); and (f) before working with the data, eliminating those swimmers with the most number of DNCs ("Did Not Compete"). Page limit prevents us from providing detailed accounts of the children's developments, however, it is important to note that the groups displayed several cycles of modelling as they worked the problem. That is, they interpreted the problem information, expressed their ideas as to how to meet the problem goal, tested their approach against the given criteria, revisited the problem information, revised their approach, implemented a

new version, tested this, and so on. We consider just a couple of the above modelling approaches in this paper.

A Focus on Personal Best Times (PBs)

Lana's group chose to focus on the swimmers' PBs from the outset, but did consider other options in justifying their decision. Initially, the group thought they might "add up the amounts," to which one member responded, "Yeah, and whoever has the smallest..." Later on, when the group revisited this option, Lana felt this was not feasible because "what I'm arguing is, well, I'm not really arguing, what I'm saying is ... we can't add up the totals because there are so many Did Not Competes, and they've got uneven amounts, so that wouldn't be fair." Another child responded, "And they would get a lot lower (total)."

In comparing the swimmers' PBs, the group members clarified their interpretation of this notion: "Don't you want the lowest time, whatever? The lowest time is the fastest swimmer." "Because that means they don't take as long to swim." As the group were considering the swimmers' PBs, they also noted an error in the data (Ashley Callus' PB was higher than his score for the 2001 Pan Pacs.) The group spent quite some time arguing about how to resolve this dilemma but decided to accept the error.

In reflecting on their focus on PBs, three of the group members questioned the reliability of these data. In the transcript below, the children are starting to think about trends in the data and swimmers' variation from their PBs.

Kelly continued her argument that "It doesn't just Kelly: Yeah, but Lana they might just one day swim really, really well, like...they might have just had a really, really good day, yeah, or week or whatever.

Lana: Yeah, I know...

Kelly: They might be a really good swimmer and then they sort of you know they might have had an injury and gone back but their not as good, so...it might have changed.

Sam: Yeah it might help to stop swimming, and like start...

Tony: What we would have to do is look at the latest times, compare those, and then we will know.

Kelly: Yeah but see in the Olympics, you don't all get into the Olympics. So, obviously they weren't...

Lana: Oh but he's saying latest times, so everyone's latest times would be in different place really...

depend on their PB, I mean you might be a really good swimmer...your PB might, like, change...because your personal best is your best but changes all the time." The group also spent time considering the PBs in relation to the level of the competition in which these were attained (e.g., a PB earned at the 2000 Olympics was more significant than one at the Telstra Australian Championships). Here the children were displaying an informal understanding of weighted variables, however, they did not pursue this further.

Children in another class were also aware of the limitations of relying solely on PBs, and began to consider possible variations from a given PB. After one group presented their report, a boy asked, “How come you just compared their personal best because they don’t do that all the time?” In subsequent class discussion the children explained that the swimmers’ most recent times should be considered, “because they could have been slow when they first started, and they could have got stronger...and now they’re going faster and faster.” The children also commented that the swimmers “could get slower and slower,” or “they could just stay on their personal best,” or “they could go faster and then slower and then fast.”

Assigning Scores and Ranking

We consider here how James’ group developed their model. First, they considered each row of Table 1 in turn, and awarded a score of 2 to the swimmer with the lowest time and a score of 1 to the swimmer with the second lowest time. They then added each swimmer’s scores and recorded these in their own table (see the first row of Figure 1). As James explained, “Some people got the most amount of points...like, some people both got first...because they need two people, so whoever came first, they would have both got two points.” Second, the group considered the PBs of each of the swimmers and awarded 2 points to the lowest PB and one point to the second lowest (see the second row of Figure 1). The group then aggregated each swimmer’s scores to find the “total rank.” Ian Thorpe and Michael Klim were thus selected.

Other groups only assigned one point to the swimmer with the lowest time in each swimming event and tallied the points to determine the two swimmers to be selected. Ashley’s group explained, “Our selection is Ashley Callus and Ian Thorpe because they both had the most winning streaks. We looked at all the competitions and we put a tally on the fastest time and then we counted them up.” Ashley’s group did not consider the swimmers’ PBs.

	Ashley Callus	Michael Klim	Eamon Sullivan	Ian Thorpe	Todd Pearson	Grant Hackett	Adam Pine
Rank	10	10	0	11	4	1	1
PB	0	2	0	1	0	0	0
Total Rank	10	12	0	12	4	1	1

Figure 1: The table created by James’ group

CONCLUDING POINTS

The present study is providing young primary school children with opportunities to develop powerful mathematical ideas and processes through mathematical modelling. A modelling problem is a realistically complex situation where students engage in mathematical thinking (beyond that of the traditional school problem) and generate conceptual tools needed for some purpose (Lesh & Zawojewski, in press). Modelling problems foster and reveal children’s mathematical thinking thus enabling teachers to capitalise on the insights gained into their children’s mathematical developments.

Table 1: Swimming Times Recorded for the Men's 100m Freestyle

Competition	Ashley CALLUS (PB: 48.43 secs)	Michael KLIM (PB: 47.98 secs)	Eamon SULLIVAN (PB 50.06 secs)	Ian THORPE (PB: 48.11 secs)	Todd PEARSON (PB: 48.45 secs)	Grant HACKETT (PB: 48.67 secs)	Adam PINE (PB: 48.68 secs)
2004 Telstra Swimming Grand Prix	51.50 secs	50.44 secs	50.35 secs	49.23 secs	50.19 secs	51.42 secs	51.87 secs
2004 Telstra Olympic Team Swimming Trials	49.31 secs	49.78 secs	50.06 secs	48.83 secs	49.78 secs	50.40 secs	50.24 secs
2003 Telstra FINA World Cup	48.06 secs	50.12 secs	50.24 secs	48.94 secs	48.83 secs	DNC	49.67 secs
2003 Telstra Australian Championships	49.07 secs	DNC	51.86 secs	49.07 secs	49.29 secs	50.32 secs	50.69 secs
2002 Pan Pacs	DNC	48.44 secs	52.43 secs	48.98 secs	49.64 secs	48.67 secs	48.93 secs
2002 Telstra Swimming Grand Prix	51.12 secs	48.58 secs	51.32 secs	48.11 secs	48.45 secs	51.90 secs	51.62 secs
2001 Telstra FINA World Cup	48.43 secs	48.43 secs	51.74 secs	48.81 secs	50.80 secs	DNC	48.90 secs
2001 Pan Pacs	47.81 secs	49.13 secs	53.73 secs	50.79 secs	50.30 secs	51.93 secs	49.46 secs
2001 Telstra Australian Championships	49.46 secs	49.53 secs	55.12 secs	49.05 secs	49.67 secs	51.69 secs	50.27 secs
2000 Telstra FINA World Cup	49.62 secs	47.98 secs	DNC	49.99 secs	48.98 secs	51.42 secs	48.68 secs
2000 Olympic Games	DNC	48.56 secs	DNC	DNC	DNC	DNC	DNC

Note: DNC means did not compete; PB means personal best

Modelling problems are designed to ensure that the product generated embodies the mathematical ideas and processes that children constructed for dealing with the problem situation. In the case of the Olympics problem, the children's creations revealed informal understandings of variation, aggregation and ranking of scores,

inverse proportion (the lower the time, the faster the swimmer), and weighting of variables—all of which are not normally part of the 4th-grade curriculum.

REFERENCES

- Diezmann, C., Watters, J., & English, L. D. (2002). Teacher behaviours that influence young children's reasoning. In A. Cockburn & E. Nardi (Eds.), *Proceedings of the 26th International PME Conference* (pp. 289-296). Norwich: University of East Anglia.
- English, L. D. (2003). Reconciling theory, research, and practice: A models and modelling perspective. *Educational Studies in Mathematics*, 54, 2 & 3, 225-248.
- English, L. D., & Lesh, R. A. (2003). Ends-in-view problems. In R. A. Lesh & H. Doerr (Eds.), *Beyond constructivism: A models and modelling perspective on mathematics problem solving, learning, and teaching* (pp.297-316). Mahwah, NJ: Erlbaum.
- English, L. D., & Watters, J. (In press.). Mathematical modelling in the early school years. *Mathematics Education Research Journal*.
- English, L. D., & Watters, J. J. (2004). Mathematical modelling with young children. In M. Johnsen Hoines & A. Berit Fuglestad (Eds.), *Proceedings of the 28th International PME Conference* (pp. 335-342). Bergen: Bergen University College.
- Erickson, F. (1998). Qualitative research methods for science education. In B. J. Fraser & K. G. Tobin (Eds.), *International handbook of science education* (Part 2). (pp. 1155-1173). Dordrecht: Kluwer Academic Publishing.
- Gainsburg, J. (2003). *The mathematical behavior of structural engineers*. Doctoral dissertation, Stanford University.
- Lesh, R., & Zawojewski, J. (In press). Problem solving and modelling. In F. Lester (Ed.), *Handbook of Research on Mathematics Teaching and Learning*. Information Age Pub.
- Lesh, R., & Doerr, H. M. (Eds.) (2003). *Beyond constructivism: A models & modeling perspective on mathematics problem solving, learning, & teaching*. Mahwah, NJ: Erlbaum.
- Lehrer, R., & Schauble, L. (2003). Origins and evolutions of model-based reasoning in mathematics and science. In R. A. Lesh & H. Doerr (Eds.), *Beyond constructivism: A models and modelling perspective on teaching, learning, and problem solving in mathematics education* (pp. 59-70). Mahwah, NJ: Lawrence Erlbaum.
- Lesh, R. A., & Kelly, A. E. (2000). Multi-tiered teaching experiments. In R. Lesh & A. Kelly (Eds.), *Handbook of research design in mathematics and science education* (pp. 197-23). Erlbaum.
- Salomon, G., Perkins, & Globerson, T. (1991). Partners in cognition: Extending human intelligence with intelligent technologies. *Educational Researcher*, 22(3), 2-9.
- Simon, M. A. (2000). Research on the development of mathematics teachers: The teacher development experiment. In R. A. Lesh & A. Kelly (Eds.), *Handbook of research design in mathematics and science education* (pp. 335-360). Mahwah, NJ: Lawrence Erlbaum.
- Stillman, G. (1998). The emperor's new clothes? Teaching and assessment of mathematical applications at the senior secondary level. In P. L. Galbraith et al. (Eds). *Mathematical modelling: Teaching and assessment in a technology-rich world* (pp.243-254). West Sussex: Horwood Publishing Ltd.