STRUCTURE AND TYPICAL PROFILES OF ELEMENTARY TEACHER STUDENTS' VIEW OF MATHEMATICS

Markku S. Hannula[1], Raimo Kaasila[2], Anu Laine[3], and Erkki Pehkonen[3]

University of Turku[1], University of Lapland[2], and University of Helsinki[3]

The elementary school teachers' view of mathematics is important because it will influence the way they will teach mathematics. Based on a survey study in three Finnish universities we explored the structure of student teachers view of mathematics and also the different belief profiles that they had. The core of student teachers' view consisted of three correlated beliefs: belief of one's own talent, belief of the difficulty of mathematics, and one's liking of mathematics. Five other beliefs were also identified. Gender, grade, and mathematics course selection at high school each contributed to the variation in these factors. A cluster analysis produced three main types of belief profiles: positive, neutral, and negative view. Each of these was further divided into two subclasses.

INTRODUCTION

Elementary school teachers' competences in and view about mathematics have a profound significance, because they are the first ones to teach mathematics to children. A negative view can seriously interfere their becoming good mathematics teachers, unless they can either overcome their anxiety or find constructive coping strategies. Fortunately, efforts to promote positive view of mathematics in elementary school teacher students have often proven at least partially successful (e.g. Kaasila, 2000; Liljedahl, 2004; Pietilä, 2002; Uusimäki, 2004).

In our research project "Elementary teachers' mathematics", our aim is to explore the belief structure of teacher students and how it develops in three universities that use different approaches to promote positive affect among students. This report will focus on revealing the structure of teacher students' view of mathematics at the beginning of their studies and identifying their typical belief profiles.

The three basic elements of the human mind are emotion, cognition and motivation. Respectively, we need to pay attention to the students' feelings, beliefs, and wants as the elements of view of mathematics. The feeling aspect of the view of mathematics consists of the emotions one experiences while doing mathematics. However, in any survey study it is only possible to find about expectancies or memories of these emotions. For the term "belief", there is no single, exact definition. Here, we shall define beliefs as purely cognitive statements to which the holder attributes truth or applicability. According to this view, beliefs do not include an emotional component, although a belief can be associated with an emotion. The aspect of motivation, or wanting, relates to the goals and desires one has. (Hannula 2004).

2005. In Chick, H. L. & Vincent, J. L. (Eds.). *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 3, pp. 89-96. Melbourne: PME.

View of mathematics has a structure. We can distinguish between one's view of different objects, such as 1) mathematics education (mathematics as subject, mathematical learning and problem solving, mathematics teaching in general), 2) self (self-efficacy, control, task-value, goal-orientation), and 3) the social context (social and socio-mathematical norms in the class,) (Op't Eynde, De Corte & Verschaffel 2002). With regard to the social context, Op 't Eynde and DeCorte (2004) found later that the role and functioning of one's teacher are an important subcategory of it. The spectrum of an individual's view of mathematics is very wide, and they are usually grouped into clusters that influence each other. Some views depend on other ones, for the individual more important views. When discussing beliefs Green (1971) uses the term 'the quasi-logical structure of beliefs' which means that the individual himself defines the ordering rules. We assume that emotions, cognitions and motivations form a system that has a quasi-logical structure. The view of mathematics also has a hard core that contains the student's most fundamental views (cf. Green 1971: the psychological centrality of beliefs; Kaplan 1991: deep and surface beliefs). Only experiences that penetrate to the hard core can change the view of mathematics in an essential way (Pietilä, 2002).

Kaasila (2000) studied the school-time memories of elementary school teacher students (N = 60). He divided the students autobiographical narratives into five groups: 1) "*It was important to be the fastest to solve the exercises*" (15 %), 2) "*Mathematics provided AHA!-experiences*" (20 %), 3) "*I survived by learning by heart*" (9 %), 4) "*Mathematics was boring, I lost my interest*" (36 %), 5) "*I fell off the track*" (20 %). Respectively, Pietilä (2002) grouped elementary school teacher students (N = 80) based on their written responses into four groups: 1) "*Mathematics is challenging problem solving*" (13 %), 2) "*Mathematics is important and usually pleasant*" (36 %), 3) "*Mathematics is one subject among others*" (20 %), 4) "*Mathematics is difficult and unpleasant*" (31 %). These two qualitative studies suggest that 20 – 30 % of Finnish elementary school teacher students have negative view of mathematics at the beginning of their studies.

The first analysis based on the Fennema-Sherman self-confidence scale indicated that 22 % of teacher students in this study had a low self-confidence in mathematics (Kaasila, Hannula, Laine & Pehkonen, 2004). Similar problems have been identified in other countries as well (e.g., Uusimaki & Kidman, 2004; Liljedahl, 2004).

METHODS

The research draws on data collected on 269 trainee teachers at three Finnish universities (Helsinki, Turku, Rovaniemi). Two questionnaires were planned to measure students' mathematical beliefs and competences in the beginning of their studies.

The 'view of mathematics' indicator consisted mainly on items that were generated in a qualitative study on student teacher's mathematical beliefs (Pietilä, 2002) it also included a self-confidence scale containing 10 items from the Fennema-Sherman mathematics attitude scale (Fennema & Sherman, 1976), four items from a 'success orientation' scale found in a study with pupils of comprehensive school (Nurmi, Hannula, Maijala & Pehkonen, 2003) and some background information about earlier success in mathematics and experience as a teacher.

The mathematical skills test contained altogether 12 mathematical tasks related to elementary level mathematics. Four tasks measured understanding of some key concepts and eight tasks measured calculation skills. The questionnaires were administered within the first lecture in mathematics education studies in all universities in autumn 2003. Students had altogether 60 minutes time for the tests and they were not allowed to use calculators.

For a principal component analysis we chose 63 items of the ‘view of mathematics’ indicator. The topic 'experiences as a teacher' was excluded, because almost half of the respondents had not had enough experience as teacher to answer these questions. We chose to use Maximum likelihood method with direct oblim rotation and examined a number of different solutions. A more detailed description of the method can be found in another article (Hannula, Kaasila, Laine & Pehkonen, 2005).

When a solution of principal components was found, the structure of the components was analysed. We calculated the correlations between components and also the effect of main background variables: gender, the selected course and grade at high school mathematics.

After extraction of principal components, a cluster analysis of the data was made in order to find different student teacher profiles. For the cluster analyses we chose 8 principal components of the student teachers' view of mathematics as the variables for clustering, squared Euclidian distance as measure, and Ward's method to determine clusters. Cluster solutions of three to seven clusters were examined.

RESULTS

Principal component analysis

In the principal component analysis we chose to use the ten-component solution, which yielded high alpha values for the components (Table 1). Two of the components had only two items loading on them and they were not included in further analysis. A full description of all components and their loadings can be found in Hannula *et al*. (2005).

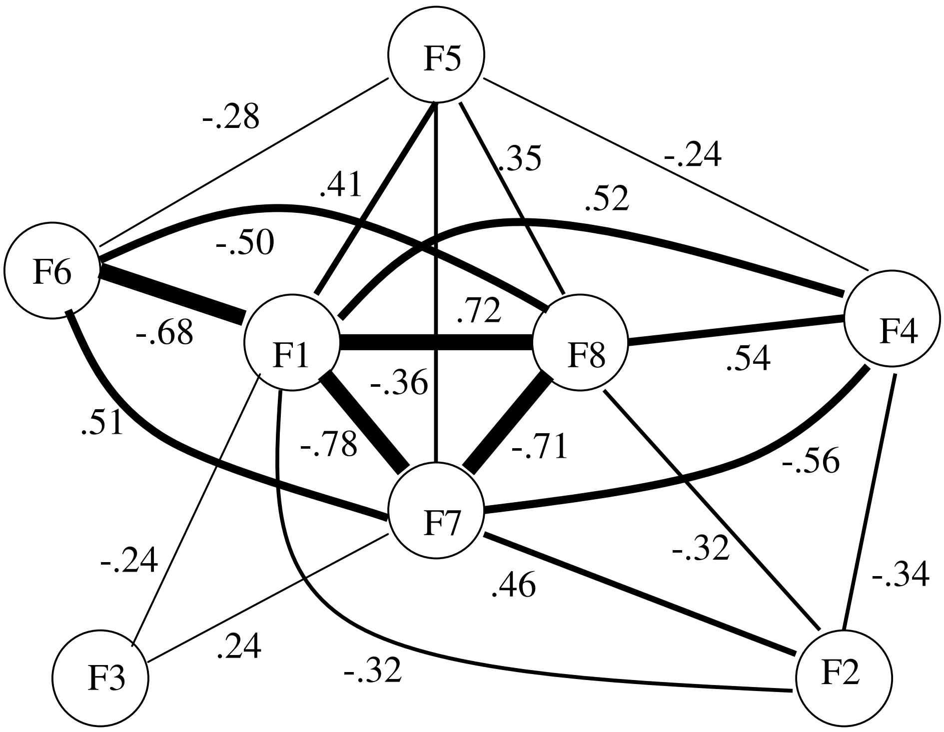
Structure of student teachers' view of mathematics

When we look at the correlations between the created components (Figure 1), we see that three of the components are closely related and form a core of the person's view of mathematics. This core consists of three aspects of person's general attitude towards mathematics. The first aspect (F1) focuses on beliefs about self, the second aspect (F8) on beliefs about mathematics, and the third aspect (F7) on the person's emotional relationship with mathematics. Around this core there are five factors, each of which relate primarily to the core and some secondarily also to each other. The encouraging family (F 3) had only a minor effect on the core view, whereas experiences of poor teaching (F 4) related more closely to the core view and also to diligence (F 2) and insecurity as a teacher (F 5). The core had also a strong connection with future expectations (F 6), which probably differs from factor 1 in that the element of effort has a greater role in it.

Table 1. The 10 principal components of teacher students' view of mathematics.

Component number	Name of the component	Number of items	Cronbach's alpha
F1	I am not talented in mathematics	8	0.91
F2	I am hard-working and conscientious	5	0.81
F3	My family encouraged me	3	0.83
F4	I had a poor teacher in mathematics	8	0.84
F5	I am insecure as a mathematics teacher	4	0.74
F6	I can do well in mathematics	4	0.80
F7	I like mathematics	8	0.91
F8	Mathematics is difficult	6	0.78
F9	Mathematics is calculations	2	NA
F10	I am motivated	2	NA

Figure 1. Structure of student teacher's view of mathematics. The connection weights are Pearson correlations. The eight factors are described in Table 1.



We found significant gender differences in most of the variables examined. The largest difference was in that female students felt that they are more hard-working and diligent (F2). Male students had higher self-confidence regarding their talent in mathematics (F1). However, there was no gender difference in students liking of mathematics (F7) or perceiving mathematics as difficult (F8). Female students had more critical image of their mathematics teachers (F4). According to a regression analyses, gender accounted for 20 % of the variation in the view of mathematics.

As assumed, the course selection in high school had affected students' view of mathematics. Those who had studied the more advanced mathematics course in high school had significantly higher self-confidence regarding their talent (F1). They also liked mathematics more (F7), but both groups perceived the subject equally difficult (F8). Those who had studied the more advanced track were less critical about their teachers (F4) and they had also received less encouragement from their families (F3). Surprisingly, the track had no effect on view of oneself as hard-working (F2), although the more advanced course is generally regarded to require a lot more work. According to a regression analyses, course selection accounts for 15 % of the variation in the view of mathematics. As there is usually a clear gender difference in course selection, we checked also for interaction effect between gender and course, but found this to be non-significant.

Those who had made it well in mathematics held more positive views about themselves and mathematics. Grade had a significant correlation with one's view of oneself as talented (F1) and hard-working (F2), as well as one's liking of mathematics (F7), and view of mathematics as a difficult subject (F8). Correlations were weaker, but still significant among female subjects with a positive view of one's teacher (F4) and one's chances to do well in mathematics (F6). All effects of grade were more pronounced among female subjects and those who have studied the more advanced mathematics course in high school. According to a regression analyses, course grade accounted for 12 % of the variation in the view of mathematics.

Cluster analysis of student views

In cluster analysis, the three-cluster solution separated students into groups based on the core of their view of mathematics: negative view, neutral view, and a positive view. In a six-cluster solution each of these was split into two subgroups, mainly based on the encouragement they got from their family and their view of themselves as hard-working (Figure 2).

A **positive view** included a view of oneself as talented in mathematics and hard-working, mathematics as easy and enjoyable, good memories of one's teachers, and confidence as a teacher and as a student of mathematics. The two subgroups of the positive view were the following:

Autonomous (21 %): These students had most positive view of all, they were rather hard-working but family had encouraged them only modestly.

Encouraged (22 %): These students had received the strongest encouragement from their family and they were the most hard-working group.

A **neutral view** fell on in the middle ground on most of the dimensions. These students had modest confidence on their own talent and they neither liked nor hated mathematics. The two subgroups of this category were the following:

Pushed (18 %): These students were encouraged by their family, but they did not work hard. They felt secure about their ability as a mathematics teacher.

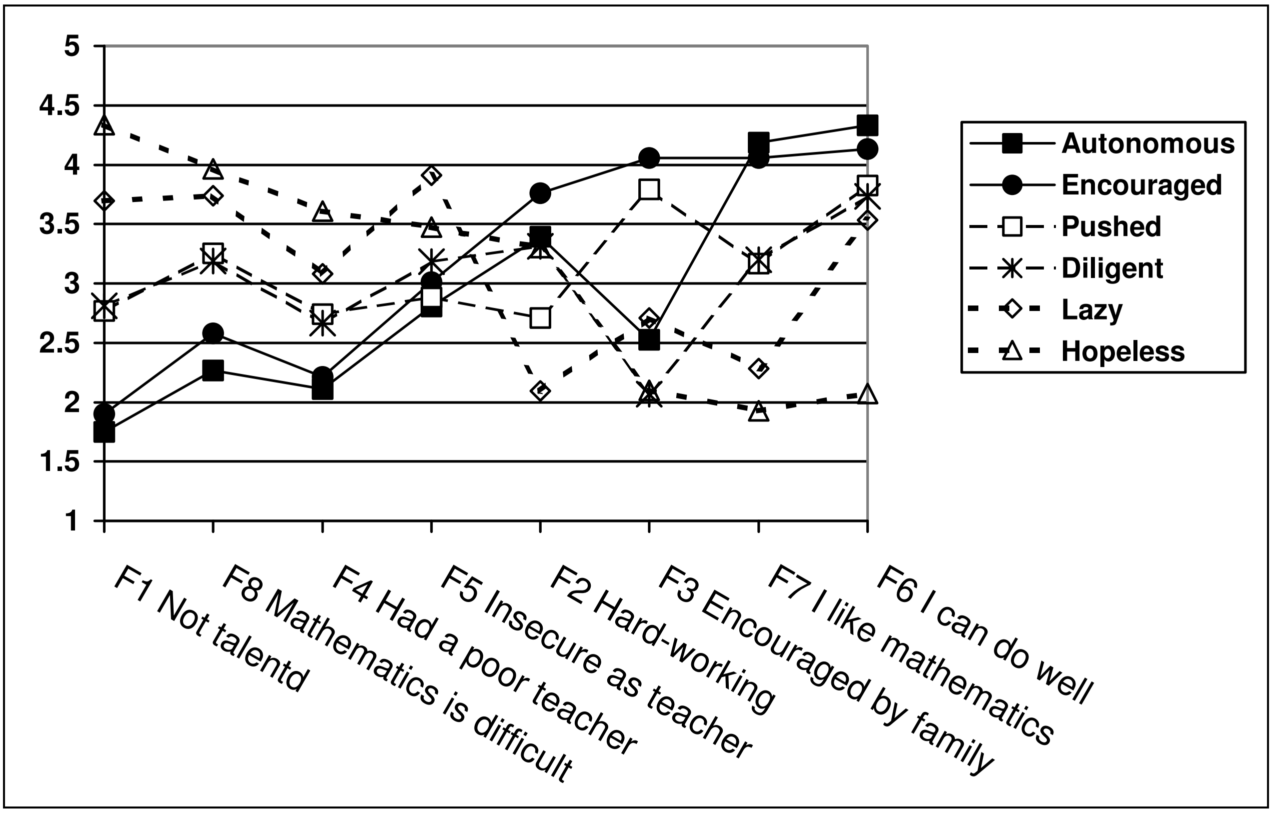
Diligent (18 %): These students were one of the two groups that received the least encouragement from their family, but they were rather hard-working.

A **negative view** included an image of oneself as not talented, dislike of mathematics, view of mathematics as difficult, negative view of one's teachers, and lack of confidence as a teacher of mathematics. The two subgroups of a negative view were the following:

Lazy (18 %): These students had a less extreme negative view compared to the other negative category. They were the least hard-working and the most insecure of all groups about teaching mathematics.

Hopeless (4 %): These students were the most extreme in their negative view. They were not encouraged by their family, yet they had worked hard. These students differed from all the other clusters in that they did not believe that they could learn mathematics.

Figure 2. The six clusters of student teachers according to their view of mathematics.



To provide a more detailed description of the clusters, we looked at how they differ in some of the other variables. We found out that students with different entrance procedure fell unevenly into different clusters. Furthermore, those who had studied the more advanced mathematics in high school were more likely to fall into the cluster of 'encouraged' students while the students who had studied the less advanced mathematics were more likely to fall into the 'diligent' cluster. Gender was not a statistically significant factor in students' distribution in different clusters, but because the number of male students was low, we can not conclude gender to be a non-significant factor either.

Clusters also differ in test results and in mathematics grades, the successful having more positive views. The clusters differ also in motivation, those with positive view having higher motivation and the 'lazy' students having lower motivation than the remaining clusters. However, the clusters do not differ in teaching experience or age.

CONCLUSIONS

Regarding the structure of beliefs, we found a core of the view of mathematics, which consists of three closely related elements: belief of own talent, belief of difficulty of mathematics, and liking of mathematics. The result supports the view that although emotion is correlated with beliefs, it is a separable aspect of one's view of mathematics.

The three background variables: gender, course selection and grade are related to many of the variables, explaining a fair amount of the variation. Interestingly, while we found gender differences in self-confidence, we did not found those in liking mathematics or perceiving mathematics difficult. Hence, it seems worthwhile to separate the different aspects of the positive view of mathematics. Female students perceive themselves to be more hard-working and diligent than male students.

Students fall within three main categories. Some have positive (43 %), some neutral (36%) and some a negative (22 %) view towards mathematics. Each category was further divided into two subcategories mainly based on how much encouragement they got from their family and how hard-working they perceived themselves to have been. Some of the students with a negative view were seriously impaired as they felt that they have tried hard and failed. Consequently, they have adopted a belief that they can not learn mathematics.

References

Fennema, E. & Sherman, J.A. (1976). Fennema-Sherman mathematics attitudes scales. *JSAS Catalog of Selected Documents in Psychology, 6*, 31 (Ms. No. 1225).

Green, T. F. (1971). *The Activities of Teaching*. Tokyo: McGraw-Hill Kogakusha.

Hannula, M.S. (2004). *Affect in mathematics education*. Annales Universitatis Turkuensis B 273. Finland: University of Turku.

Hannula, M. S., Kaasila, R., Laine, A., Pehkonen, E. (2005). *Student teacher's view of mathematics at the beginning of their studies*. A paper accepted to CERME4 conference.

Kaasila, R. (2000). *"Eläydyin oppilaiden asemaan" Luokanopettajaksi opiskelevien kouluaikaisetn muistikuvien merkitys matematiikkaa koskevien käsitysten ja opetuskäytäntöjen muotoutumisessa*. ["An Insight into the role of Pupils." The significance of school recollections in the formation of the conceptions and teaching practices of mathematics for preservice teachers.] Acta Universitatis Lappoensis 32.

Kaasila, R., Pehkonen, E., Hannula, M.S., & Laine, A. (2004). *Pre-service elementary teachers' self-confidence in mathematics at the beginning of their studies*. A paper presented in TSG23 at ICME-10. Available online at http://www.icme-organisers.dk/tsg23/tsg23_abstracts/rTSG23014Kaasila

Liljedahl, P. (2004). *AHA!: THE effect and affect of mathematical discovery on undergraduate mathematics students*. A paper presented in TSG3 at ICME-10. Available online at http://www.icme-organisers.dk/tsg03/TSG3_Liljedahl.pdf

Kaplan, R.G. (1991). Teacher beliefs and practices: A square peg in a square hole. In R. Underhill (ed.), *Proceedings of the annual meeting of the North American chapter of the international group for the Psychology of Mathematics Education, Vol 1*, pp. 119-125. Blacksburg, VA.

Nurmi, A., Hannula, M., Maijala, H., & Pehkonen, E. (2003). On pupils' self-confidence in mathematics: gender comparisons. In N.A. Pateman, B.J. Dougherty & J. Zilliox (eds.) *Proceedings of the 27th Conference of the International Group for the Psychology of Mathematics Education. Vol 3,* 453-460. University of Hawaii.

Pietilä, A. (2002). *Luokanopettajaopiskelijoiden matematiikkakuva: Matematiikka-kokemukset matematiikkakuvan muodostajina.* [Pre-servise elementary teachers' views of mathematics: The role of mathematics experiences in forming the views of mathematics.] University of Helsinki, Department of Teacher Education, Research Reports 238.

Op 't Eynde, P., De Corte, E. & Verschaffel, L. (2002). Framing students' mathematics-related beliefs: A quest for conceptual clarity and a comprehensive categorization. In G. Leder, E. Pehkonen & G. Törner (eds.). *Beliefs: A hidden variable in mathematics*. Pp. 13-37. Dordrecht: Kluwer.

Op 't Eynde, P.& De Corte, E. (2004). *Junior high students' mathematics-related belief systems: Their internal structure and external relations*. A paper presented in TSG24 at ICME-10. Available online at:
http://www.icme-organisers.dk/tsg24/Documents/OptEyndeDeCorte.doc

Uusimaki, S.L.M & Kidman, G. (2004). *Challenging maths-anxiety: An intervention model.* A paper presented in TSG24 at ICME-10. Available online at http://www.icme-organisers.dk/tsg24/Documents/UusimakiKidman.doc