

ONE TEACHER'S ROLE IN PROMOTING UNDERSTANDING IN MENTAL COMPUTATION

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This paper reports the teacher actions that promoted the development of students' mental computation. A Year 3 teacher engaged her class in developing mental computation strategies over a ten-week period. Two overarching issues that appeared to support learning were establishing connections and encouraging strategic thinking.

While a growing interest in mental computation as a vehicle for developing number sense has become a focus in many international mathematics curricular (e.g., Maclellan, 2001; McIntosh, 1998; Reys, Reys, Nohda, & Emori, 1995), mental computation is new to the Queensland (Australia) scene. In fact, many schools in Queensland have not introduced mental computation into their mathematics programs to date, as the new syllabus (Queensland Studies Authority (QSA), 2004) will not be mandated until the year 2007. However, some schools have been keen to embark on the development of mental computation. Certainly, text book writers have been quick to publish new mathematics texts that include mental computation exercises. The student books provide practice for students to apply particular strategies they have been taught. Often, the focus is on one or more specific strategies; therefore, the students practise the strategies, rather than engage in the thinking involved. This often results in a routine approach to teaching mental computation. In reality, it is easy to see why text books could become popular in the teaching of mental computation, as teachers often do not have the knowledge to sequence and present worthwhile mental computation activities.

In the context of this study, mental computation refers to efficient mental calculation of two- and three-digit addition and subtraction examples. Mental computation does not refer to the calculation of number facts. This is in contrast to the discussion of mental computation in the new syllabus (QSA, 2004), where mental computation strategies for Levels 1 and 2 (relevant to the children in this study) refer to basic facts strategies (e.g., count on, count back, doubles, near doubles, make to 10). Even Level 3 'mental strategies' do not include strategies that have been identified elsewhere as appropriate for young children to develop, for example, *compensation (N10C)* (e.g., Beishuizen, 1999; Thompson, 1999).

At present in Queensland (Department of Education, Queensland, 1987), children in Year 3 (approximately 8 years of age) are expected to be able to complete addition and subtraction two-digit with and without regrouping and three-digit without regrouping written algorithms. The final product is generally procedural with little understanding.

One school that has embarked on the development of mental computation (in the early years – Years 1–3) is the one described in this paper. For the purposes of this paper, only the work conducted in the Year 3 class will be discussed. In 2004, the researcher worked with the Year 3 teacher to develop a program to enhance mental computation. This teacher had also been involved in a similar study in the previous year (reported in Heirdsfield, 2004a, b). The previous year's work impacted on the present study, as the teacher had already been introduced to the literature on mental computation; conducted some pre-interviews with her students to establish their base knowledge; plan a mental computation instructional program in conjunction with the researcher; and, then, implement the program. The researcher acted as a critical friend. Finally, the teacher conducted some post-interviews to identify growth in students' mental computation, measured by strategy choice and accuracy; and reflected on the project; for instance, identification of effective models (e.g., empty number line, hundred chart), sequencing, and questioning; and level of student participation and interaction. Therefore, the teacher already had some knowledge about what constituted an effective mental computation program.

Several research studies investigating successful instructional programs (e.g., Blöte, Klein, & Beishuizen, 2000; Buzeika, 1999; Gravemeijer, Cobb, Bowers, & Whitenack, 2000; Hedrén, 1999; Kamii & Dominick, 1998) have indicated that the emphasis of instruction should be strategic flexibility and students' exploring, discussing, and justifying their strategies and solutions. In addition to student behaviour, teacher competence is also an important factor in successful instruction (e.g., Askew, 1999; Brown, Askew, Baker, Denvir, & Millett, 1998; Brown, Askew, Rhodes, Denvir, Ranson, & William, 2001; Brown & Campione, 1994; Diezmann, Watters, & English, 2004). Summarising these studies, important factors in effective teaching include teacher expectations, instruction as systemised and connected, and the four teaching characteristics of Brown et al. (2001) – tasks, talk, tools, and relationships and norms. Therefore, teacher competence is a key factor in students' quest for understanding.

The purpose of the project was to enhance Year 3 students' mental computation performance. The specific aims were to collaboratively design an instructional program to build on students' existing strategies, and to identify and monitor students' mental computation performance. The instructional program was based on students' prior knowledge (identified from individual interviews). This paper focuses on the identification of teacher actions that promoted the development of mental computation.

THE STUDY

The research adopted a case study design (Yin, 1994) in which a teaching experiment (Steffe & Thompson, 2000) was conducted with the aim of developing Year 3 children's mental computation performance. The study was conducted in a Year 3 class (7-8 year olds) consisting of 30 students, in a school serving a predominantly

middle class community in an outer suburb of Brisbane. Students engaged in 30 to 45 minute lessons once a week for 10 weeks. These lessons focussed on the development of mental computation strategies for 2- and 3-digit addition and subtraction. The teacher and researcher had worked together in the previous year on a similar project, when the teacher was

A similar approach was taken in 2004. In addition, it was decided that teaching of the traditional pen and paper algorithm (which is still used in Queensland schools) would be avoided for the duration of the project. Pre- and post-interviews were conducted by the researcher, teacher and a research assistant. The teacher incorporated learning from the previous year into the instructional program. Each lesson was videotaped; and observations (including comments) of the lessons were documented by the researcher. The focus was on identifying the connections and sequencing of the lesson, student participation and communication, the sense that students were making during the lesson, questioning, and quality of interaction, in general. The researcher was a participant observer, who interacted with individual students and small groups during the lesson. Each lesson was followed by a brief discussion between the teacher and the researcher, where clarification of the aims and perceived outcomes was sought, and inhibiting factors and avenues to pursue were identified. The teacher was also provided with a copy of the researcher's notes for further consideration, and as a record of the lesson from an observer's view. The videotapes were later analysed for further insight. Data comprised videotaped lessons, the researcher's field notes, student work samples, the teacher's lesson plans and reflections, and the pre- and post-interviews. Data were analysed to identify emerging issues related to the students' mental computation reasoning.

RESULTS

Analysis of the teacher's actions revealed two issues that influenced student mental computation performance. Well planned questioning; provision of appropriate tasks and models; a great deal of exploration, discussion, and critiquing of strategies; and careful sequencing were used to establish connections and encourage strategic thinking.

Connections

From the previous year's project, the teacher became aware of the importance of sequencing both within a lesson and between lessons. The researcher formulated a suggested sequence for introducing number combinations in conjunction with appropriate models (empty number line, hundred chart, & 99 chart):

1. jumping in tens forwards and backwards from multiples of ten (e.g., start with 40 – jump forwards or backwards in tens);
2. jumping in tens forwards and backwards (e.g., start with 43 – jump forwards or backwards in tens);

3. relate the previous step to addition and subtraction (e.g., start with 43 – add 10, add 20, add 30; take away 10, take away 20, etc);
4. further addition and subtraction, without bridging tens (e.g., 43 ± 22);
5. further addition and subtraction, bridging tens (e.g., 47 ± 28 ; 47 ± 19 as a special case). For an example of the type 47-28, only the ENL might be used, as it supports a *going-through tens* strategy (Thompson, 1997). A hundred chart cannot easily be used for this strategy (for subtraction); although, a 99 chart can be used.

Progress through steps one and two were easily completed in one lesson, but progress to step three, for some students, required making the connections explicit. The teacher successfully scaffolded these students learning with careful questioning.

Start at 33 (on the hundred chart) and jump to 53. How far is that?

Some students responded with “twenty” and others responded with “two tens”. Both responses were accepted. For others who were hesitant, a further line of questioning was pursued.

Start at 33 (on the hundred chart). Add 10 more. Where are you now? Where did you start? What did you add on? Now add another 10. Where are you now? Where did you start? What did you add on altogether?

As well as the teacher scaffolding the slower students, class discussion was encouraged. Students who originally were hesitant started to make connections by participating in this discussion.

To do 66 and 20 more, I said that’s the same as ten and ten more.

I said that’s the same as two tens.

By the time, students were presented with examples of the type at steps 4 and 5, the teacher documented students’ strategies using equations, as they explained their strategies; for instance,

86-45		
$86-5=81$	or	$86-40=46$
$81-40=41$		$46-5=41$

Although the students had viewed this documentation several times, there was no smooth progression to the students’ documenting their own strategies in the same way. So, the students were placed in small groups, made up of a recorder, demonstrator and speaker. The recorder (who documented the equations) and speaker (who was to present the strategies to the class) had to listen very carefully to the demonstrator while the strategy was being described and check that all steps had been documented. The researcher and teacher scaffolded many groups through this process. However, success was achieved (see Figure 1).

A final example of making connections concerns the use of the empty number line. In Queensland, students have had no experience with the empty number line: although, now, some teachers are using this model. The teacher introduced the empty number line by firstly providing the students with number lines where tens were labelled and divisions between tens marked (see Figure 2).

$$43 - 25 = 18$$

$$43 - 20 = 23$$

$$23 - 3 = 20$$

$$20 - 2 = 18$$

$$57 + 39 = 96$$

$$57 + 30 = 87$$

$$87 + 10 = 97$$

$$97 - 1 = 96$$

$$43 - 25 = 18 \quad 43 - 20 = 23 \quad 23 - 10 = 13 \quad 13 + 5 = 18$$

$$72 - 39 = 33 \quad 72 - 30 = 42 \quad 42 - 10 = 32 \quad 32 + 1 = 33$$

Figure 1. Examples of students' written documentation of strategies

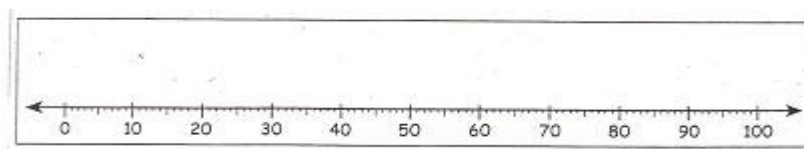


Figure 2. Number line used to introduce the empty number line

In addition, the teacher used a white board drawn up with number lines where tens were marked. A large clear plastic sheet sat over the whiteboard, so that jottings on the plastic sheet could be removed without affecting the number lines drawn on the whiteboard. While the students worked on their number lines, the teacher and individual students worked on the number line on the whiteboard. The students were directed to find/mark numbers on their number lines, and explain how they knew how to find the numbers. They then jumped on from or backwards from these numbers in tens. Finally, the connections were made between jumping in tens and adding and subtracting multiples of tens (e.g., $73-40$). Again, scaffolding questions were required for some students.

Start at 33 (on the number line). Add 10 more. Where are you now? Where did you start?
 What did you add on? Now add another 10. Where are you now? Where did you start?
 What did you add on altogether?

The empty number line was introduced by the need to use a more flexible number line. The teacher drew a straight line (with no markings) on the blackboard, and the example " $95+30$ " was written above the line. Discussion was opened up to the class to decide how best to use the number line to solve the problem. One student suggested placing 95 towards the right of the line "because that's where 95 is".

However, others suggested that the line would then need to be extended to permit the calculation to be recorded. One student suggested placing 95 to the left end, to permit the jumps to the right to be completed. The remainder of the class agreed with this solution. That student was then invited to draw the solution on the empty number line. There was also discussion about possible solutions – some suggested jumping in tens; while some suggested they could jump 30 in one go. From there, steps 4 and 5 (see above) were followed for the empty number line.

Strategic thinking

While students were introduced to models (hundred chart, ninety-nine chart, empty number line) to aid the development of mental computation strategies, the focus was not the models, but the strategic thinking. Therefore, students were free to choose any model (or no model) to solve the examples. Further, they were constantly encouraged to explain their reasoning, compare their own strategies with others' strategies, and critique the strategies. Apart from a means of solution, the models were also used as a means of communication. Sometimes, when the students discussed their strategies, the teacher documented the strategies using the models, and sometimes she documented the strategies in equations. Further, students were permitted to use any model or no model. In fact, during two lessons, students were provided with a page of empty number lines (it was found that students wasted precious time if they drew their own number line – they were obsessed with using rulers), and a sheet with a hundred chart and a ninety-nine chart. The teacher presented the students with examples to solve, and individual students were invited to present their own examples for the class to solve. They were permitted to use any model (but were asked to identify the model that they used) or no model if they chose to work solely in their head, and they were asked to explain their strategy.

Students decided that the number combinations often determined (for them, individually) what model they might use. For instance, to solve $47+26$, some students preferred the hundred chart, as a *going-through tens* strategy could easily be employed. However, others preferred the empty number line for the same reason. In contrast, to solve the subtraction example $64-28$, the ninety-nine chart was preferred by some, again because the *going-through tens* strategy could be employed; while, others preferred the empty number for the same reason. When three-digit examples were presented, for instance, $192-28$, some students suggested constructing hundred and ninety-nine charts that covered these numbers; while others suggested the empty number line was more appropriate, because of its flexibility. By this stage of the project, however, some students were beginning to solve examples without models, and using strategies that did not reflect the support of models.

I did $99+47$ by saying, that's the same as $100+47$, but then took one away.

CONCLUDING COMMENTS

The focus in this teaching experiment was not merely on developing mental computation strategies, but on higher order thinking – reasoning, critiquing, engaging in sense making, both in what they did and in what they said. The teacher suggested that there were higher participation rates and enthusiasm on the part of the students compared with previous mathematics lessons. Strategic thinking was encouraged, rather than merely “getting the right answer”. The teacher reported that in other number work students were exhibiting a sense of number – they were talking about numbers in more flexible ways and making more sense of computations. When students were reintroduced to formal written algorithms (after the completion of the teaching experiment), they made sense of the algorithms – rather than merely following procedures. The students were making connections.

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