

LEARNING AND TEACHING EARLY NUMBER: TEACHERS' PERCEPTIONS

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The context of this study is the current curriculum reform in South Africa. Teachers' didactical knowledge is regarded as playing a crucial role in students' learning and this study investigated teachers' perceptions about ways in which children learn number skills and concepts. Six Foundation Phase teachers from schools in the Cape West Coast Winelands region of South Africa were involved in the study. The data was collected by means of stimulated-recall interviews based on constructed classroom vignettes and teachers' comments were analyzed in terms of accepted theories on learning and acquisition of number. The study revealed that teachers have a limited understanding of how children learn number and gives support to the idea that a learning pathway description may assist with broadening understanding of learning and teaching early number.

INTRODUCTION

The background to this study is the reform movement in mathematics education in South Africa over the past 10 years, the launch of Curriculum 2005 (C2005) in 1997 and the Revised National Curriculum Statement (RNCS) in January 2004. The introduction of the new curriculum meant that some 1400 registered syllabi (Chisholm, 2001) had to make way for the creation of a core national curriculum that aims at an integrated system of education and training and a single national qualification framework while attempting to address issues of inequality, principles of multiculturalism and notions of citizenship in South Africa (Young, 2001).

A key feature of C2005 was the shift in emphasis from traditional subject-based knowledge and skills, to the statement of broader overarching outcomes. The formal presentation of content was removed and teachers were expected to plan their own instructional sequences. Teachers' didactical knowledge necessarily played a crucial role in the implementation of the new curriculum and approach to teaching mathematics. This required a good understanding of conceptual development and progression of content. Ultimately this feature of C2005 'disabled' many teachers who already had difficulty selecting and teaching appropriate content. The RNCS specifies content to be covered for each grade in the form of minimum standards. Understanding teachers' knowledge is necessary for effective professional development and in-service support for the implementation the new curriculum and improved teaching and learning. The purpose of the study reported here was to gain information about teachers' knowledge and their perceptions of learning and teaching

early number. Number-related strands in the RNCS Mathematics Learning Area form the basis of all learning in mathematics at the Foundation Phase and receive the greatest learning and teaching attention and time allocation.

CHARACTERIZATION OF THE STUDY

An exploratory quantitative study was undertaken with six Foundation Phase (Grade R – 3) teachers from thirteen rural and peri-urban primary schools who were interviewed in order to elicit their understanding of the ways in which young children learn number concepts. The discourse that teachers use when describing their knowledge and understanding of number acquisition in the early grades was of particular interest. For the data collection, stimulated-recall interviews were used, based on four constructed classroom vignettes and related open-ended questions. The vignettes and questions were based on current theories on learning and number acquisition and were constructed in such a way that they could act as prompts to encourage teachers to reflect on, and express their opinions about, certain aspects of learning and teaching number in different contexts.

THEORETICAL FRAMEWORK

A review of research and programmes on early number (Kühne 2004) engaged with Cognitively Guided Instruction (CGI) (e.g. Carpenter, Fennema, & Franke, 1996), the Problem Centered Primary Mathematics Program (PCM) (Murray, Olivier, & Human, 1998), Realistic Mathematics Education (RME) (e.g. Van den Heuvel-Panhuizen, 2001a), Mathematics Recovery (e.g. Wright, 1994) and Count Me In Too (e.g. Stewart, Wright, & Gould, 1998). These programmes have in common a focus on helping teachers understand the mathematics of specific content domains and children's mathematical thinking in those domains. Each programme operates from a perspective that teachers' knowledge and understanding of children's mathematical thinking is a critical factor in supporting children's mathematical learning. These programmes and other literature associated with learning and teaching early number were significant for this study as very little systematic research on early number acquisition exists in South Africa. The theoretical framework used for the analysis in this study drew on Steffe's (1992, 2000) model which outlines four basic counting schemes or stages, and on the notion of emergent counting as used by Wright (1998). The framework was founded on the idea that the advancement through these stages begins from the ability to invent informal context-related solutions, to the creation of various levels of efficient solution strategies (short cuts) and schematisations, and the acquisition of insight into the underlying principles and the discernment of even broader relationships (Van den Heuvel-Panhuizen, 2001a).

The framework identifies the following stages for learning early number:

1. Emergent Counting
2. Perceptual Stage
3. Figurative Stage
4. Initial number Sequence Stage
5. Tacitly-nested Number Sequence
6. Explicitly-nested Number Sequence Stage.

In addition to the hierarchy of stages, the framework contains an explicit description of behaviour which can be expected of most children at each stage and an overview of children's knowledge, strategies and solutions to number problems.

METHOD

Instruments

A stimulated-recall interview method (Dunkin et al., 1998) was used to elicit data. This required research participants to respond to hypothetical learner and classroom episodes. The interview instrument developed for the present study contained four parts. Below is an example of one vignette used in the interview.

Interview 1: Part 1 (I-1)

A class is investigating addition by using counting rods. The learners are working in pairs. The teacher asks one learner in each pair to represent and write the number 34, and the other learner to represent and write the number 27. The learners are then asked to combine the rods and tell her how many there are altogether. She asks several pairs to explain how they found their answer. She then demonstrates a method for recording what the learners have done with the rods.

Please consider the following questions:

1. What do you think the teacher is hoping to achieve from this activity?
2. What number concepts do you think are being developed?
3. How do you think learners will represent the numbers?
4. What method do you think the teacher would have demonstrated?

The vignettes and related questions were intended to give participants an opportunity to discuss particular features of children's number development, elicit information about their knowledge and understanding of the developmental processes underpinning the learning of number and the didactics they use when mediating this learning in the classroom. The four interview parts (I-1, I-2, I-3 and I-4) include several stages of number development (see Table 1).

Stages	I-1	I-2	I-3	I-4
Emergent Counting		*	*	*
Perceptual Stage		*	*	*
Figurative Stage	*	*		*
Initial Number Sequence stage	*	*		*
Tacitly-nested Number Sequence	*	*		*
Explicitly-nested Number Sequence Stage	*			*

Table 1: Stages across interviews

Vignette 1 was designed to encourage discussion about knowledge of number structure, basic calculation skills and strategies necessary to understand and successfully solve problems involving numbers up to 100. Important aspects of the discussion were expected to include: the sensory activity of counting-all that is still

needed for some children (Figurative stage); the notion that children have constructed numerical counting concepts and schemes and use counting-on strategies (Initial number sequence stage); the idea that they have developed ways of keeping track of their counting acts (Tacitly-nested number sequence stage); an understanding that children apply part-to-whole reasoning and have an awareness of addition and subtraction as inverse operations (Explicitly-nested number sequence stage). In order to check the validity of the interview instrument the vignettes were validated and piloted prior to use.

Sample

Six teachers from schools in the Cape West Coast Winelands region of South Africa participated in this study. Four participants in this study taught at Afrikaans medium primary schools located in a small peri-urban area. One teacher taught at a small Afrikaans medium primary school situated on a wine farm and another at a large Xhosa medium school situated in an informal settlement outside a small rural town. The number of children in the Foundation Phase classes at these schools ranges from 40 to over 70. All the teachers involved in this study were women, each of them had taught at the primary level for more than 10 years, and each had experience teaching Grade 1.

The highest formal qualification held by three teachers was a senior school certificate (matriculation) plus three additional years of tertiary training (M+3). Two teachers had completed a matriculation certificate and had studied for a further four years (M+4). One teacher received her teacher training qualification prior to completing her matriculation certificate.

Data collection and analysis

The data was collected in the second term of 2003. Participants were first asked to complete the teacher questionnaire in order to establish accurate teacher profiles. Each participant was interviewed once; each interview lasting between one and two hours. The analysis of the interview data was conducted by one of the authors of this paper working both deductively and inductively in developing a theoretical framework. Initially she worked with the transcripts deductively, using the elements of the theoretical framework to describe and classify teachers' comments according to *elements of the stages in number development*. She also worked more inductively with the data in order to identify participants' comments on *other didactic themes* related to learning and teaching number.

RESULTS

The first part of the analysis disclosed how often the participants (P1-6) reflected knowledge of stages in number development (Table 2) and made references to other didactic themes (Table 3) in their comments to the vignettes and questions when they were interviewed.

Stages	Number of times participant reflected knowledge of stages in number development						
	P1	P2	P3	P4	P5	P6	Total
Emergent Counting			1			1	2
Perceptual Stage	2		1				3
Figurative Stage	4					1	5
Initial Number Sequence Stage	3		2	1	1	1	8
Tacitly-nested Number Sequence	4	5			1	7	17
Explicitly-nested Number Sequence	1		3				4
Total	14	5	7	1	2	10	

Table 2: Results related to elements of the stages in number development

The analysis of stages in number development revealed that the participants in general reflected an understanding of number development. However, they did not mention specific stages, levels or categories as included in the theoretical framework. References to the early stages of number development – emergent numeracy, perceptual stage and figurative stage – were mostly contained in participant remarks in relation to weak number concepts or poor learner performance. The following quotes are examples of comments which illustrate this point:

..can't write any number or show to a number or even count out. He can count without understanding, he's counting like the parrot but he doesn't understand what he is really doing. (P3:63– 65)

The one with the poor concept of number must start from the beginning, he must every time have the picture from the beginning again and you have to tell him start in 3's from 3 to 18, then he'll say 3, then he'll count on his fingers to get to the 6 and maybe count again to get to the other one, but he won't off-hand be able to count (P1: 205-209)

Comments which related to the initial number sequence stage, tacitly-nested number sequence and explicitly-nested number sequence stage were made in relation to performance and skills which participants considered most desirable. The majority of comments pertained to the tacitly-nested number sequence. In particular, as the following examples illustrates, that automated number facts are more desirable than counting based strategies.

And then Freda is also using a long method because she is counting with her hands. Zondi is also using the long method. Lisa is using a little bit a short method because she didn't use a lot of coins to come up with that 12 (P6: 46-48)

Ayanda was very fast! Ayanda didn't use any concrete objects to come to that 12. Fran was also a little bit ...very...he was also fast because he just count 5 and 4 is 9 plus 1 is 10 and then 2 is 12. Ayanda was marvellous! ..the one that is going to take a loooong method and the one who is just counting and tell you, even when you doing some demonstration on the board just NO, they are calling me dadobawo (aunt), dadobawo that is 8 or that is 7 (P6: 49-54)

A large number of references were also made to the initial number sequence stage. The following quote highlights a discussion regarding addition strategies:

But if they going to put it together they'll also just, if they clever enough, the one will take the 34 and they'll just count on, just add on the 27 to get to the new number. (P1: 24-26)

The second part of the analysis showed that the participant comments include four other didactic themes; included the teachers' role, sequencing material, perceptions of good and weak learners and the effect of environment on learning and teaching (Table 3).

Didactic Themes		Number of times participant referred to other didactic themes in number development						
		P1	P2	P3	P4	P5	P6	Total
Teachers' Role	Direct instruction	1	2			3	3	9
	Indirect instruction	3	3		2	1	1	10
Sequencing Material	Concrete to abstract	1	1	1	4			7
	Representation	1			1		4	6
	Range	6	1	1	2	3	3	16
	Prior-knowledge	3	2	2	3	3	3	16
	Planning by age/time	1	1			3	3	6
Perceptions of good and weak learners		12	3	2	3	1		21
Effect environment on learning/teaching				1	2	3		6
Total		28	13	7	17	21	17	

Table 3: Results related to other didactics themes in number development

The comments indicated that participants considered various ways of supporting number development, such as organizing and sequencing material and learning in particular ways. Participants' comments also revealed an awareness that understanding and acquisition of skills are linked to various forms of representations and that this is a structured process that develops gradually from the use of concrete manipulatives to more abstract symbolic forms of (mental) representation. An example is this comment which indicates a strong position on the use of concrete manipulatives as a pre-requisite to understanding number in its related abstract forms:

I suggest is that you should use concrete things to learn a child. He must use the real thing to count, we can't just count and we can't just do sums. Every time we do a sum when it comes to number you must give the child a problem, because if we just work from, if the child just see that number on the board or I just give them the number and he put out the sum, and there isn't a problem attached to it, he will never understand, he will never be able to make a, be a critical thinker when it comes to maths. Because if they understand number, they will be able to understand everything else in maths, because it starts with number. (P1: 245-253)

All participants in this study suggested that learners should be provided with a range of learning experiences that facilitate the move from concrete to abstract forms of representation, and that these should include pictorial, concrete and symbolic forms. Most participants' comments indicated an awareness of the idea of progression. This

progression was expressed in terms of the use of manipulatives, building on prior-knowledge and experience and ways of organizing learning according to time-related aspects (including age or grade). The analysis of comments further revealed that participants recognize certain factors that impact on learners' performance. These factors include teaching styles and the role of the teacher in facilitating or supporting learning, and the effect of environments on children's learning.

CONCLUDING REMARKS

This study has a number of limitations, such as not including classroom observations and not interviewing teachers in their mother tongue, therefore the results should be treated with prudence. Furthermore, the limited number of teachers involved does not allow wider generalizations. The study showed that the participating teachers have a limited understanding of how children learn number and the complexity and diversity of this process. It also revealed that they understand children's number development in particular ways, which is expressed within a framework of their own classroom experience. Teachers do not share a common discourse about number development. It is also evident from the analysis of teachers' comments that they do not describe an overview of how number develops or what can be expected in the learning process from a particular understanding.

Although teachers in this study recognize that learning should be organised and sequenced in particular ways, this was mainly described in terms of notions of concrete to abstract representations. Discussions about sequencing number concepts progressively were not based on cognitive development but rather on generic teaching approaches. Teachers' did not discuss the process of modeling concepts and operations which enable children to bridge the gap between informal, context-bound strategies and formal standardized operations. From this study it does not appear that teachers are able to describe a long-term overview of the process of learning and teaching number that connects the different development stages and offers a framework for didactical decision making. These findings are perhaps surprising given that all teachers had attended in-service mathematics courses. This suggests the need for a more elaborated form of in-service education with a framework or trajectory for learning and teaching number, such a trajectory would present mathematical understanding and content in a progressive and structured way and encompasses the cognitive and didactic continuum. A trajectory that emphasises relationships between various forms of representation, highlights different learner strategies and 'bridges' the RNCS and classroom practices would be a useful tool to assist in pre and in-service teacher education. This instrument would also support the arduous task of selecting and sequencing content and classroom activities.

Recently a research and development project, inspired by the Dutch TAL Learning-Teaching Trajectory for calculation with whole numbers (Van den Heuvel-Panhuizen, 2001b), was initiated which aims to develop and measure the impact of a Learning Pathway for Number (LPN) at the Foundation Phase.

References

- Chisholm, L. (2001). *History and Contemporary Politics of Curriculum Construction and Development in South Africa: Presented at the 'Oxford' International Conference on Education and Development*, 9-11 September 2003 at the session on Culture, Context and the Quality of Education.
- Carpenter, T.P., Fennema, E., & Franke, M. (1996). Cognitively Guided Instruction: A Knowledge Base for Reform in Primary Mathematics Instruction, *Elementary School Journal*, 97(1), 3-20.
- Dunkin, M., et al. (1998). Teachers' explanations of classroom events: Knowledge and beliefs about teaching civics and citizenship. *Teaching and Teacher Education*, 14, 141-151: National Academic Press, Strategies Education Partnership, Washington, United States.
- Kühne, C. (2004). *Teachers' Perceptions of Whole Number Acquisition and Associated Pedagogy in the Foundation Phase*. Unpublished Masters Dissertation (Teaching). University of Cape Town: South-Africa
- Murray, H., Olivier, A., & Human, P. (1998). Learning Through Problem Solving. In A. Olivier & K. Newstead (Eds.), *Proceedings of the 22nd Conference of the International Group for the Psychology of Mathematics Education* (Vol 1, pp. 169-185). Stellenbosch, South Africa: University of Stellenbosch.
- Steffe, LP. (1992). Learning Stages in the Construction of the Number Sequence. In J Bideaud, C Meljac & J Fischer (Eds.) *Pathways to number: Children's Developing Numerical Abilities* (pp. 83 – 88), Hillsdale, NJ: Lawrence Erlbaum
- Steffe, LP. (2000). *Principles and Standards for School Mathematics (PSSM) From a Constructivist Perspective*, paper written for the Research Symposium Linking Research and the New Early Childhood Mathematics Standards held at the 2000 National Council of Teachers of Mathematics (NCTM) Research Precession, Chicago, Illinois
- Stewart, R., Wright, B., & Gould, P. (1998). Kindergartener's progress in the Count Me in Too project. In C. Kanen, M. Goos, & E. Warren (Eds.), *Teaching mathematics in new times (Proceedings of the 21st annual conference of the Mathematics Education Research Association of Australasia)*, (pp. 556-563). Gold Coast, QLD: MERGA.
- Van den Heuvel-Panhuizen, M. (2001a) Realistic Mathematics Education in the Netherlands. In J. Anghileri (Ed.), *Principles and Practices in Arithmetic Teaching: Innovative Approaches for the Primary Classroom* (pp. 49-63). Buckingham/Philadelphia: Open University Press.
- Van den Heuvel-Panhuizen, M. (ed) (2001b). *Children Learn Mathematics: A Learning-Teaching Trajectory with Intermediate Attainment Targets for Calculation with Whole numbers in Primary Schools*, Freudenthal Institute, Utrecht University, Utrecht
- Wright, R.J. (1998). *Children's Beginning Knowledge of Numerals and It's Relationship To Their Knowledge of Number Words: An Exploratory, Observational Study*, Psychology of Mathematics Education: Stellenbosch, South Africa
- Wright, R.J. (1994). *A study of the numerical development of 5-year-olds and 6-year-olds*, *Educational Studies in Mathematics*, 26, 25-44, Kluwer Academic Publishers, Dordrecht, The Netherlands
- Young, M. (2001). *Education Reform in South Africa, 1990-2000: An International Perspective*. Human Sciences Research Council, Pretoria, South Africa.