

THE TRANSITION OF A SECONDARY MATHEMATICS TEACHER: FROM A REFORM LISTENER TO A BELIEVER

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Research on professional development has focused on elementary school teachers. This study is part of a larger study that investigates professional development strategies that support the growth of secondary teachers. Findings indicate that secondary teachers may need experiences that challenge them to re-examine their knowledge and identity before considering implications of reform mathematics.

Mathematics educators (Farmer, Gerretson, & Lassak, 2003; Harel & Lim, 2004; Kazemi & Franke, 2004; Lin, 2004; Patterson & Norwood, 2004) investigated several professional development strategies to deepen teachers' content and pedagogical knowledge. Farmer, Gerretson, and Lassak found that teachers developed their content knowledge when they solved non-routine problems and reflected on how they could be used in their classrooms. By reflecting on the learning environment, these teachers noticed critical aspects in the learning environment that promoted their own learning and were motivated to change facets of their own practice. Kazemi and Franke found that when teachers asked their students to solve the same task and then discussed their students' work during a professional development session, the teachers attended to the details of students' thinking and began to create learning trajectories to develop more sophisticated reasoning. Lin found that helping teachers create situations for students to pose-problems provided student responses for assessment and instructional planning. These studies indicate that helping teachers become more aware of how their knowledge and actions influence students' learning appears to be critical for teachers' professional growth.

Mason (2002) describes the art of noticing as "being awake to situations, being mindful rather than mindless" and can be cultivated through deliberate acts (p. 38). Researchers (Heinz, Kinzel, Simon, & Tzur, 2000; Kazemi & Franke, 2004) suggest that teachers' professional growth is linked to their ability to listen carefully to students' articulation of mathematical ideas, to ask probing questions, and to consider students' responses in light of mathematical concepts. The following five strategies supported elementary teachers to notice details of students' responses: (a) using rich tasks (Stein, Smith, Henningsen, & Silver, 2000), (b) using a research-based framework that maps the development of number sense (Carpenter, Fennema, Peterson, & Carey, 1988), (c) asking questions (Haydar, 2003), (d) examining students' responses (Lin, 2004; Harel & Lim, 2004), and (e) prompting teachers' pedagogical curiosity (Olson, in press).

This study extends previous research by investigating the complexities of deepening teachers' content knowledge simultaneously with their pedagogical knowledge. It is part of a larger research project which investigates how different professional development approaches influence teachers' beliefs and actions in the classroom. Specifically, this study sought to describe how a course in which secondary teachers created conceptual rubrics to interpret students' responses and plan instruction influenced their beliefs and practices.

THEORETICAL FRAMEWORK

The theoretical framework guiding this study is that of situated learning in which knowledge is co-produced as individuals discuss, adapt, or create models (Boaler, 2000). The situated perspective assumes that learners function in a social context and that learning can not be isolated within a class. Learning is located within classrooms, schools, and communities and the practices in these setting influence how an individual incorporates new ideas into their beliefs and practices.

Research using a perspective of situated learning focuses on how individual develop and use knowledge through their interactions within a social context (Boaler, 2000). To investigate how individuals create new understandings in a social environment, Lave (1993) suggested that researchers collect and interpret data that reflects an individual's learning through his or her actions. This data can then be interpreted to indicate changes in an individual's competence and knowledge, identity, and practices.

METHODS

Six secondary and seven elementary teachers to investigate rational numbers using the textbook *Teaching Fractions and Ratios for Understanding* (Lamon, 1999) during a Master's Degree course taught by Olson. To create collaborative groups, the thirteen teachers answered three opened-ended questions (a) How do students show you that they understand math? (b) Are students born smart in math or do they become smart? (c) What can teachers do to help struggling students become successful? These responses were interpreted as an indication of each teacher's beliefs about teaching and learning. Olson assigned the teachers to one of four groups, comprised of both secondary and elementary teachers who held different views of learning. During each class period, the teachers shared their solution strategies for assigned problems, investigated problems from the textbook, and discussed research that supported constructivist learning theories. Teachers identified underlying mathematical concepts for the grade-level rational number benchmarks. Then, the teachers described student behaviors that indicated a developing, basic, or proficient level of understanding for each underlying concept. These levels of understanding were condensed into a developmental rubric that was used to create a series of lessons that might advance students' reasoning about rational numbers.

A high school teacher (second author) agreed to participate as a case-study to investigate the changes of her beliefs and practices as she collaboratively worked with two elementary and one secondary teacher during the class. Olson made field notes detailing the group's interactions and collected written reflections and solution strategies. Kirtley created a proportional reasoning rubric and designed a series of lessons to enhance students' ability to use proportional reasoning to solve problems.

After completing the course, Kirtley redesigned these lessons using a lesson plan format suggested by Smith (State Mathematics Conference, 2004) that focused her attention on eight aspects: (a) Goals, (b) Previous knowledge, (c) Solution strategies, (d) Engagement, (e) Expectations, (f) Introduction, (g) Questions, and (h) Conceptual understanding. Kirtley recorded her reflections, observations about her students, and conceptions about teaching and learning in a journal. She analyzed these data for her Master's Degree Project (Kirtley, 2004) and described the process of her professional growth.

Olson's field notes, Kirtley's written work, and Kirtley's Master's Degree Project were analysed for changes in Kirtley's attention. These changes were interpreted as evidence that Kirtley was noticing new details as she interacted within a social environment. Matrices were utilized to reduce and synthesize data to describe facets of professional development which influenced Kirtley's competence, identity, and practice.

RESULTS AND DISCUSSION

Before beginning a Master's Degree, Kirtley taught in an urban high school with a history of low achievement, poverty, and high truancy. She described herself as a "savvy high school teacher" who was a strong mathematician and passed the teacher licensure content exam with a high score (Kirtley, 2004, p. 5). As such, she considered herself to be an expert and continually used mathematics books as references while preparing lessons (interview, January 5, 2005). Kirtley wanted students to understand "why" but frequently became frustrated with their low performance. Kirtley explained, "I didn't believe that students could do it [high level math] because they didn't do it after I showed them how. I didn't do all the activities in the reform curriculum because I considered many of them to be fluff." (Kirtley, 2004, p. 13). These comments reveal a core belief that mathematics is best taught through procedures and that students' demonstrated understanding when they arrived at a correct answer.

Kirtley's competence, identity, and practices reflect Stigler and Hiebert's (1999) description of traditional high school teachers. She described hearing the reform message, students need to understand mathematical concepts before procedures can make sense, repeatedly though out the Master's program but they were only words that did not fully resonate with her own experiences (Kirtley, 2004).

Mathematical competence and identity

Kirtley's small group collaboratively solved non-routine problems using manipulatives and pictures before linking their strategies to a symbolic representation. By the second session, Kirtley was angry. She wrote, "I wondered why anyone would spend time and effort to go into fractions this way [solve problems using multiple representations] in a way that was quite unfamiliar to me" (Kirtley, 2004, p. 11). Kirtley revealed a core belief about learning when she asked the class to consider the role of procedures, "Isn't it important for students to just know their facts and quickly multiply $\frac{2}{3}$ by $2\frac{1}{4}$ without using pictures?" (class discussion, June 15, 2004) Olson responded, "The question is not the importance of procedural knowledge but how and when it occurs. When students understand a concept first, the procedure makes sense and you don't need to continually repeat instruction to gain mastery. Let's take a look at trying to understand the procedure for dividing fractions."

The class reviewed two representations for $8 \div 2$. Twelve teachers interpreted division using a measurement model, eight items were placed in groups of two forming four groups. One student used the partitive model, eight items were placed in two groups with four in each group. Olson asked the collaborative groups to represent $1\frac{1}{2} \div \frac{1}{3}$ using the measurement and partitive models of division. Kirtley's group represented $1\frac{1}{2}$ with a circle and a semi-circle. An elementary teacher (El Teacher 1) began the discussion.

El Teacher 1: The circle is the unit, so one and a half circles represents $1\frac{1}{2}$. To divide by $\frac{1}{3}$, we need to find how many groups we can make of $\frac{1}{3}$. (The group divided the circle into thirds and the half circle into a third and a left over piece.) We have 4 groups of $\frac{1}{3}$ and a sixth.

Kirtley: But when I divided $1\frac{1}{2}$ by $\frac{1}{3}$, I got $4\frac{1}{2}$.

El Teacher 2: Uhm, let's look at the sixth. It is a sixth of the whole circle but only half of a group of $\frac{1}{3}$. So, we have 4 groups of $\frac{1}{3}$ and half of a group of $\frac{1}{3}$, that would be $4\frac{1}{2}$.

Kirtley: I know how to do this with numbers, but I don't get these pictures. I'm starting to buy into the idea that making pictures help you understand the math, but why do kids need to be able to draw pictures?

El Teacher 2: If you draw a picture then you can see what's going on.

Olson: Try to represent the problem by interpreting the problem as, you have $1\frac{1}{2}$ and it is in a third of a group. How many would be in one whole group?

Kirtley: I don't know if I can think of it that way. This is really hard.

Olson: In the problem, what is one-third of a group?

El Teacher 1: One and a half?

Olson: What do you think (to the other teachers)?

Sec Teacher: Yes, I have to write down your questions, but if $1\frac{1}{2}$ is a third of a group, then we have to have 3 groups of $1\frac{1}{2}$, the whole group.

Olson: Can you draw a picture to show it?

El Teacher 1: Yes (draws one and a half circle, labels it $\frac{1}{3}$ and repeats it 2 more times.)
When I count up the pieces, there are $4\frac{1}{2}$ circles. It is right!

Kirtley: And what you drew is what I'd do with the procedure. You invert the $\frac{1}{3}$ and multiple. That's what you did with the picture. You made 3 groups of $1\frac{1}{2}$. I can't do this by myself. The elementary teachers understand how to make representations with pictures and I need their help.

Olson: So your group is learning from each other. The elementary teachers need help from you [the secondary teachers] to connect the pictures to the algorithms.

In this excerpt, the four teachers worked collaboratively to create two different representations for dividing fractions. One elementary teacher drew a picture using the measurement interpretation but struggled to interpret a symbolic representation for the remaining fractional piece. Kirtley used symbols to solve the problem but could not relate the mathematical procedure to the picture. When the group created a representation using a partitive interpretation, Kirtley quickly noticed a connection between this representation and the standard algorithm. She recognized that elementary teachers have a deep understanding of representing mathematical ideas that she lacked and there was more to learn. With this realization, she devised additional problems to solve using pictorial representations. Kirtley later reflected, "I didn't get representing fractions and was upset. Then, it turned me on. I loved seeing multiple representations and how everything connected. I was amazed how deeply you could go into a seemingly simple subject. All students struggle with fractions and I felt empowered to teach. This class was the first time that I experienced math as a student and from that perspective saw the importance of deep understanding." (Kirtley, 2004, p. 12).

Kirtley's mathematical competence was challenged when asked to create a pictorial representation and a new facet of mathematical thinking emerged. In the collaborative problem-solving setting, Kirtley engaged in a mathematical activity in a novel way and her identity shifted as she recognized her reliance on the elementary teachers for creating pictorial representations. Kirtley experienced mathematics in a new way and identified that this experience was critical for her growth.

Teaching practice

After completing the course on rational numbers, Kirtley continued to reflect about the intersection of her beliefs, practice, and students' learning. She described herself as "a changed teacher; I needed to help my students have the same profound and exciting feelings about math that I experienced that summer" (Kirtley, 2004, p. 11). Changing her beliefs about teaching and learning occurred after she struggled to solve problems without using a mathematical procedure which challenged her competence and identity. With this struggle, Kirtley described a fundamental change, "I really believe that my students can learn and that everything I've heard over the

past two years [while working on her Master's Degree] fit together and made sense" (reflective notes, September 2004).

Kirtley described two events during her final semester in the Master's Degree Program that cemented her belief in conceptual understanding: attending a State Mathematics Conference and a school district seminar for mathematics teacher leaders. At the conference Kirtley listened to Margaret Smith's presentation on "Thinking Through a Lesson Protocol" (TTLP) and Jeremy Kilpatrick's presentation on the five mathematical proficiencies (September 24, 2004) and these presentations reinforced her new conceptions about teaching and learning.

Kirtley used TTLP to plan 20-activities and noticed that students were more engaged when they created representations to solve problems than when she "skipped the activities" and showed them symbolic procedures (Kirtley, 2004, p. 13). Students used the manipulatives until they "became a liability when the figures became more complex" and created symbolic representation themselves to keep track of data (reflective notes, November 2004, p. 23). For example, Kirtley's students investigated the relationship between the area of triangle and its base and height by constructing all possible triangles with an area of two on a geoboard. Student groups made posters to display all possible triangles but struggled to provide a rationale to prove that their list was complete. Kirtley asked a series of reflective questions like, "Could you make a triangle that was three units long on the base? One unit long? Four units long? Explain." (reflection, November 2004, p. 34). Eventually, her students explained that a triangle had an area of two only if a rectangle could be constructed around it with an area of four. Kirtley noticed a relationship between the cognitive demand of a question, students' mathematical thinking, and wait time. She reflected, "I felt very happy with the students for thinking ... I gave a difficult question, and waited for students to answer."

Kirtley was invited to attend the school district's seminar for Mathematics Teacher Leaders. She felt excited about the seminar and the opportunity to learn more. Kirtley wrote, "Some teachers wanted to be leaders in their school but were bitter about being asked to think conceptually. I had great empathy for them, because just one year ago, I was one of them... As I sat in the seminar, a wonderful feeling came over me. I knew I had changed in a way that was exciting and empowering." Kirtley, 2004, p. 14).

Kirtley deepened her conceptual understanding of mathematics, developed a new identity of herself as a learner of mathematics with a unique voice, and changed her teaching practices to build students' conceptual understanding before discussing more efficient strategies. These changes occurred after struggling in a class which provided an opportunity to develop her own mathematical understanding in a collaborative group of teachers and reconsider her beliefs in light of these experiences. Kirtley's professional growth continued as she interacted with students in her classroom and with teachers and experts in seminars and conferences.

CONCLUSIONS

Research (Carpenter et al., 1988; Haydar, 2003; Lin, 2004; Stein et al., 2000) indicates that professional development that deepens elementary teachers' content and pedagogical knowledge by examining tasks, questions, and developmental frameworks can support teachers' growth. Olson initially thought that secondary teachers might demonstrate growth using these same professional development strategies. But, in this study, the professional growth of a highly competent high school teacher was prompted by cognitive dissonance in a social setting.

The collaborative problem-solving environment disrupted Kirtley's identity as a competent, self-confident teacher to confront herself as a student struggling to use mathematical ideas in a new way. She discovered that she needed the help of elementary teachers who were more adept at modeling to be successful. As her competence at solving problems with pictorial representations grew, she recognized how the symbols represented her actions and understood the reform message in a new way. The process of change began when Kirtley's competence and identity were challenged. Kirtley wanted her students to feel the same excitement in the fall and sought follow-up support to cement her new beliefs about teaching and learning into practice.

This study suggests that professional development that supports growth for high school teachers may be different from elementary teachers. The interaction between high school teachers and elementary teachers with their different expertise was critical to help a traditional high school teacher re-examine her own content knowledge and identity. Further research is needed to describe professional development that supports the growth of high school teachers and to investigate whether collaboration between elementary and secondary teachers is a viable professional development model.

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