TEACHER FACTORS IN INTEGRATION OF GRAPHIC CALCULATORS INTO MATHEMATICS LEARNING

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Graphic calculators (GC) have been widely used in teaching for at least 15 years, and yet many teachers still appear to be unaware of their potential for learning mathematics. This research describes a study addressing aspects of the teacher's role in integrating GC technology into their pedagogical approach. It considers issues of didactic contract, pedagogical technology knowledge, procedural and conceptual knowledge in GC integration of 7 secondary teachers. The results describe a number of factors that identify teacher progress toward pedagogical integration of the GC.

BACKGROUND

Brousseau (1997) has described the mutual recognition of the parts that the student and teacher play in the classroom learning process as a *didactic contract*. This term recognises, through the tacit acceptance of each party, that there are *reciprocal obligations* in the relationship. Not least among these is the expectation by students that they will be taught, and by the teacher that the students will want to learn. Of course this social contract is a dynamic entity, changing and adapting to new circumstances that arise in the classroom milieu. Factors influencing a teacher's didactic contract include affective variables (beliefs and attitudes), perceptions of the nature of mathematical knowledge and how it should be learned, mathematical content knowledge, and pedagogical content knowledge (Shulman, 1986). One may assume that if a teacher possesses a limited knowledge of a concept and its related subconcepts (Chinnappan & Thomas, 2003) then they will find it more difficult to provide the kind of environment and experiences that will assist students in the construction of rich conceptual thinking. Instead they may regress to a *process-oriented* approach (Thomas, 1994), presenting students with a toolbox selection of procedures that may be applied to each problem that arises. While such procedures and skills are important, mathematical thinking is clearly much wider than this, and requires procedural and conceptual interactions with the various representational forms of mathematics (Thomas & Hong, 2001). However, teaching is not mediated simply by the mathematical understanding of the teacher (Cooney, 1999), but it is also influenced by the teacher's pedagogical content knowledge. This refers to understanding the mathematical ideas involved in a particular topic and how these relate to the principles and techniques required to teach and learn it, including appropriate structuring of content and relevant classroom discourse and activities (Shulman, 1986; Simon, 1995; Cooney, 1999; Chinnappan and Thomas, 2003).

2005. In Chick, H. L. & Vincent, J. L. (Eds.). *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 4, pp. 257-264. Melbourne: PME.

The introduction of new technology into the classroom has been shown to be capable of a subversive effect (Thomas, Tyrrell, & Bullock, 1996) radically altering the didactic contract. Thomas, Tyrrell, and Bullock (*ibid*. p. 49) suggest that the introduction of technology requires a new mindset on the part of teachers, a '*shift of mathematical focus'*, to a broader perspective of the implications of the technology for the learning of the mathematics. It has become clearer working with teachers in the years since this study that the *technology knowledge* aspect of instrumentation (Rabardel, 1995), namely how to control the functioning of the tool, is insufficient for a successful mathematics outcome. In addition teachers need to develop what we now call *pedagogical technology knowledge* (PTK), knowing how to teach mathematics with the technology. This arises as they progress through the stages of instrumentalisation and instrumentation of the tool (Rabardel, 1995), gaining a personal appreciation of its role in learning mathematics, and importantly, of ways in which students may be assisted through various teaching approaches to emulate their instrumentalisation and instrumentation of the technological tool.

As instrumentation of the technological tool proceeds teacher beliefs and attitudes are shaped changing their teaching emphasis, and didactic contract, to give increased emphasis to the instrument. In turn, student preferences have been shown to mirror this *teacher privileging* (Kendal & Stacey, 2001). As teachers progress in their belief in the value of the technology in teaching mathematics they have to face the key issue of the level of integration of the technology in learning that they will espouse. This may range from using it at prescribed moments as a teacher-directed add-on, to an ever-present instrument that is an extension of cognition. The research described in this paper followed a group of seven teachers as they began, or continued, their instrumentation of the GC tool, thus extending their PTK. It attempted to understand teacher practice in relation to the congruency of content knowledge, pedagogical content knowledge, instrumentation, PTK and didactic contracts.

METHOD

During August 2004 a GC professional development workshop was arranged over three weeks (3 sessions of 2 hours each) for teachers from two Auckland schools. The course covered both content and pedagogy for algebra and calculus, using the TI–83Plus with several downloadable memory-based FlashApp[lication]s. The teachers who volunteered to attend the workshop had little experience of using the GC to teach mathematics, although two had previously used them. Four teachers attended the workshop from school A, and initially six from school B, but only three of these finished the course, including one trainee teacher. Apart from this trainee, all the teachers were experienced, with between 10 and 24 years teaching. Each teacher was given their own TI–83Plus GC and class sets during the workshop and they kept these for six months after the course. Once the teachers were familiar with the TI–83Plus they asked questions relating to their teaching, and discussed ideas with one another. In the month following the workshop the teachers were given a brief questionnaire on their perspective of the value of the TI–83Plus for teaching mathematics, and all four teachers from school A and two teachers from school B agreed to take part in the classroom-based phase of the research. During this phase we were able, over a three week period of teaching years 10–13 (age 15-18 years), to observe and video their classroom teaching with the GC. They also completed a diary of teaching with TI–83Plus detailing the mathematical content covered and their aims and objectives. The videotapes were transcribed for analysis along with the data from the questionnaire, the lessons and the diaries. Following discussion, the topics the teachers chose included families of curves, linear programming, limit, drawing graphs and derivatives.

RESULTS AND DISCUSSION

One of the first things that a teacher new to using CAS, or indeed any other technology, has to decide is how they will structure its role in their classroom. This is a basic feature of changes to their didactic contract. All the teachers in the study organised their classrooms in a similar manner. They chose to have the students sit in traditional rows and the teacher spent some time at the front of the class, demonstrating examples using a viewscreen while the students followed and copied their working onto their own calculator. This may have been in order to maintain control of the classroom situation. Afterwards the students spent the rest of the time working on problems and tackling exercises as a group, while the teachers circulated and assisted with any difficulties. In spite of this similarity in approach it was soon clear that the teachers were different in the pedagogical advances that they made with the technology. Three of them made good strides forward, while two proceeded more cautiously, and two made little advancement. An analysis follows of some of the differences we perceived between the groups in terms of the variables described above, based on one teacher exemplifying each of the three groupings.

Little advancement

A major factor here influencing the PTK of teacher E from school B, was his lack of confidence with the GC, springing from a lack of instrumentation. He had previously used a scientific calculator in his teaching, and saw value in the GCs, commenting that “Several topics are made ‘easier’ with a GC. But students need to have their own.” He particularly singled out content areas of “Linear programming, simultaneous equations (3 variables), graphing of level 2 graphs” and thought GCs “Very good with ongoing learning. Inequations helpful”. However, when asked if he had problems with the GCs he answered “Yes, lots—need to spend more time using them regularly.” and he found them “Time consuming to learn processes and to remember these processes.” He was conscious of still being in the early stages of instrumental genesis. In class he chose to use the GC to teach year 12 (age 17 years) students “graphing of parabolas, cubics, exponential function, hyperbola, graphing inequalities, linear programming, and solving simultaneous equations” in up to 3 variables. Asked what problems his students faced he said “Same problems. I need to be continually using them or I forget", indicating that like him they often forgot the correct commands, and showing where his emphasis lay. This is a crucial aspect of the instrumental genesis of the GC tool and leads to a lack of confidence in teaching. Evidence of this was observed in the lessons he taught. We can see from statements during a lesson on linear programming, such as, "I think you just have to go 3 down, and push ENTER", "Now what you probably need is… what I've done wrong here" and "I'm not sure what the calculator is going to do because we don't have anything written here", that he was not too confident in his handling of the GC commands, as he freely admitted after the lesson. This lack of familiarity with the operational facets of the GC means that a teacher such as this tends to be tied to the mechanics of operating the tool and due to this heavy cognitive load cannot free up enough thinking space to concentrate on the mathematics. This can often lead to a very procedural, button-pushing emphasis in the lesson, as we observe from the typical quote below.

> So, if you change it from ALPHA F3, we've got X=25, Y=50. I think now, it's going to go around the vertices. We went ALPHA F3, and we were able to move around, and we went ALPHA F3 again, ALPHA F4 initially then ALPHA F3. By doing that we were able to go around.

Cautious progress

In contrast teacher F, a trainee teacher from school B, made cautious progress in her growth of PTK, using the GC in her teaching of mathematics. Teacher F expressed that she "Would like to use [the GC] in teaching" but the potential advantages were seen in a procedural light as "Seems easier in some ways to sketch graphs etc", and when asked if she had problems using the GC she replied "Some difficulty". She taught year 13 linear programming with the GC, but in her interview again focussed on procedural matters as a motivation stating that "I thought it would be a good idea to show different ways for calculating unknowns in linear programming." When asked what she thought it was important for her students to learn she said "Different ways of obtaining the same answers." And that she would introduce the ideas by "By providing examples and working through these." Discussing why students might find the GC helpful she did not refer to mathematical ideas but thought it would be "By providing an interesting way to find solutions." She explained that she believed students "Need a step-by-step explanation." when using the GC, and that their main difficulty would be "Losing track of where they were." So while she did not express a lack of confidence her teaching approach was firmly set in a step-by-step, process- and solution-oriented mode, focussed on the GC rather than the mathematics. This seems to be a feature of this transitional stage of progress in the acquisition of PTK. A second feature of this stage is the inability to take the mathematics and adapt the technology to focus on the content under study.

Teacher F simply took the ideas that were presented in the workshop and tried to present these to her year 13 students. Her instrumental genesis was still very much in progress as shown when she was asked a question early in the lesson, during memory clearance. A student asked ”How do you get rid of inequalities?” and the reply, indicating lack of knowledge, was “Oh, that’s what we are going to putting them on, so don’t worry.” However, she did not concentrate as much on button pushing as teacher E, and so was able to exhibit more occasions when mathematical ideas, even if basic, were encouraged to surface, such as graph intersections corresponding to equation solutions, and testing vertices to find a maximum.

> So, ok, so at the moment we got one that’s a bit we can’t see, so we want to have it shaded where the intersections the graphs are only. So next we will press ALPHA F1 and we want to press 1 for an equation intersection. Press 1 and wait and this could be shading out from the intersections.
>
> So now what we want to do is, we want to record all our intersections for the vertices, because we are working out the maximum, and we know that when we are working out the maximum we want to put all values of vertices into that expression, so you see this one here says POI-trace and that can trace the intersection point, so press ALPHA F3.

However, there was no investigation or discussion of these, or any other, concepts, and, as we see below, the focus was clearly on obtaining the right answer to the problem and then proceeding to the next one.

> Did you get the maximum of 17?…Ok, so this is how we found the maximum so our answer is 17 which is when x=1 and y=5. So I’ll just give you an example to try yourself. That should say 1 and 5. Anyway I’ll give you an example today. So following the instructions and directions on the sheet try doing this question.

Greater strides forward

Teacher A will be used to exemplify the group that had made greater strides forward. She had over six years’ experience of using GCs in her teaching and had been involved in a previous research study with them. In spite of her experience, in the questionnaire she admitted “Sometimes it’s hard to see how to use it effectively so I don’t use it as continuously as I should.” Her motive was a rather pragmatic “We should move with the times” and she had a small reservation about the GC that “It is OK. By now expected better resolution though.” Due to her relative experience she appeared confident in her use of the GC and spoke at length in her interview, describing how “In the past I have also done some exploratory graphs lessons where students get more freedom to input functions and observe the plots.” Further, she explained that she was happy to loosen control of the students and let them explore the GC and help one another: “Students learn a lot by their own exploration…In past lessons I have never had a student get lost while using a graphics calculator. Sometimes friends around will assist someone” However, she acknowledged her need to progress in her types of GC use, “I would like to see them used more frequently and beneficially in class with structured lessons.” While she could perceive imaginative GC usage, part of her difficulty involved the pressure of day-to-day teaching, since she admitted that “I was not relaxed enough with the term coming to an end and other aspects in this year’s teaching to be inspired to use the calculator with imagination for the students." Responding to what she wanted her students to learn she replied in terms of the challenge of the depth of mathematics, "The success for me as a teacher is when they want to learn more and students show a joy either in what they are doing or in challenging themselves and their teacher with more deeper or self posed mathematical problems." She was convinced that the novel and challenging nature of the GC could motivate students—"The calculator puts a radiant light in the class… With a graphics calculator lesson no one notices the time and no one packed up."—and that her perspective on learning mathematics influenced her PTK came out in the comment that "Today we find a lot of Maths does not need underlying understanding… I feel as teachers what we need to really be aware of is what the basics are that students must know manually… when we sit down to work with graphics calculators we need to consider carefully what still should be understood manually." In this way she addressed an important factor of the integration of technology use into mathematics, namely learning what is better done by hand and what could be done better with the technology (Thomas, Monaghan, & Pierce, 2004).

One of her lessons with the GC was with Year 12 students and she considered families of functions with the aim of exploring exponential and hyperbolic graphs and noting some of their features, "we're going to utilise the calculator to show that main graph and then we're going to go through families of $y=2^x$". She was comfortable enough to direct them to link a second representation "Another feature of the calculator I want you to be aware of..[pause] you've got also a list of x and y values already done for you in a table." Teacher A had moved away from giving explicit key press instructions, instead declaring "I want you to put these functions in and graph them and see what's going on.", and "You can change the window if you want to see more detail, and if you want to see where it cuts the x-axis, you can use the "trace" function." Figure 1 shows a copy of her whiteboard working.

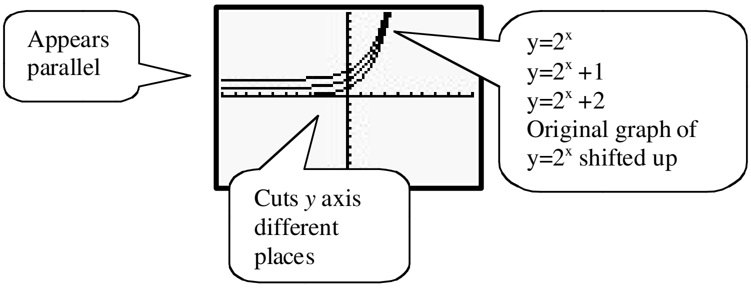

Figure 1: Teacher A's whiteboard working: Viewscreen projection and overwriting.

She was also able to move towards an investigative mode of teaching "if you're not sure where the intercepts are, you can use the "trace" key, remember, and I want you to observe what is happening.", encouraging students to use the GC in a predictive manner, to investigate a different family.

> We want to do some predictions… Looking at the screen try to predict where 3×2^x will go then press "$y =$…" and see if it went where you expected it to go. You may get a

> shock… Can you predict where "$y = 4 \times 2^x$" will be? Now you learned from that, so can you predict where it'll lie. The gap between them gets smaller. If you're interested put in "$y = 100 \times 2^x$". Does it go where you expect?

There was also some discussion of mathematical concepts and how this could help with interpretation of the GC graph. She linked 2×2^x with 2^{x+1} and then during examination of the family of equations $y=2^x$, $y=2^{x+1}$, $y=2^{x+2}$ said of $y=2^{x+1}$ "We expect this to shift 1 unit to the left [compared with 2^x]. Did it?" In this way she made a link with previous knowledge of translations of graphs parallel to the x-axis, and then reinforced this with the comment that "With this family, when you look at the graph can you see that the distance between them stays the same because it's sliding along 1 unit at a time. The whole graph shifts along 1 unit at a time." In addition, there was a discussion of the relationship between the graphs in the family of $y=2^x + k$, and the relative sizes of 2^x and k.

> … as the exponential value gets larger, because we're adding a constant term that is quite small, it lands up becoming almost negligible. So, when…all they're differing by is the constant part, you'll find that they appear to come together. Do they actually equal the same values ever? Do they ever meet at a point? No, because of the difference by a constant, but because of the scaling we have, they appear to merge.

The discussion on the relative size of terms in the function continued with "How significant is "+1" or "+2"? We know that 2^5 is 32." and again the use of prediction was evident "I want you to predict where $y=2^x + 3$" would be."

In summary we may describe the differences in the progress of the teachers we have observed in terms of a number of variables that delineate two clearly different groups, with a third progressing between the two. The first group may be identified in terms of their instrumental genesis as teachers who are still coming to grips with basic operational aspects of the technology, such as key presses and menu operations. This leads to a low level of confidence in terms of teaching with the GC in the classroom. In terms of their PTK, this group is characterised by an over-emphasis on passing on to students operational matters, such as key presses and menu operations to the detriment of the mathematical ideas. Furthermore, the mathematics approached through the technology has an emphasis on technology, and work tends to be very process-oriented; based on procedures and calculating specific answers to standard problems. There is little or no freedom given to students to explore with the GC, and it tends to be seen as an add-on to the lesson rather than an integral part of it. These features then become part of the teacher-initiated expectations in the didactic contract.

In contrast to this, the second group have advanced to the point where they are competent in basic instrumentation of the GC and are thus more able focus on other important aspects, including the linking of representations such as graphs, tables, and algebra, and to use other features of GCs. In turn, this better instrumentation of the GC produces a higher level of confidence in classroom use. Considering their PTK they begin to see the GC in a wider way than simply as a calculator. They feel free to loosen control and encourage students to engage with conceptual ideas of mathematics through individual and group exploration of the CAS, investigation of mathematical ideas, and the use of prediction and test methodology. For these teachers the mathematics rather than the technology has again been thrust into the foreground, and the GC has been integrated into the lessons and forms part of the didactic contract. If we think that the approach of this second group is preferable, then we must ask how we assist teachers to progress towards it. One answer is by the provision of pedagogically focussed professional development, relevant classroom focussed resources and good lines of teacher-researcher communication.

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