POSTER PRESENTATIONS

WORKING WITH MATHEMATICAL MODELS IN CAS

Cand. Scient. Mette Andresen

Danish University of Education

CENTRAL BOX WITH THE KEY QUESTIONS

According to experiences gained by the participants in the development project 'World Class Math & Science' computer use in upper secondary school mathematics has certain potentials. The key question in the Ph.D. project is:

-How could these potentials be identified, captured and conceptualised?

Initial inquiries and studies led to the hypothesis: Introduction of the new construct of a conceptual tool denoted 'flexibility' is a suitable conceptualisation of the potentials.

The suitability of this conceptual tool was evaluated on the background of the Ph.D. project's objective of changes. A subproject introducing the modelling approach to differential equations was chosen for inquiry of the questions:

- Is 'flexibility' a supportive construct for articulation of experiences from teaching and learning within a modelling approach? For realisation of the learning potentials of students' concept formation within this approach?

SURROUNDING TEXT BOXES

Setting: The World Class Math and Science project. Laptops in math and science for upper secondary school.

Flexibility: Background, foundations and definition. Also in http://www.icme-10.dk/index.html

Changes: 1) At curriculum level: Change from a structural point of view on differential equations to a dynamic, modelling viewpoint. 2) In the use of models and modelling: From a functional perspective of 'applied math' to inclusion of a concept formation perspective. 3) At the level of teachers' professional development: Articulation of the teachers' tacit knowledge.

Methodology. Interpretative approach, using a teaching experiment design with classroom observations etc., analysed from an 'emergent perspective' (Kelly&Lesh)

Example of data analysis. Excerpt from transcription of video recordings of students work in a small group, followed by analysis of the episode. Interpretation in terms of flexibility of how an 'emergent model' was established and negotiated

Conclusion

The notion of flexibility is useful to structure the analysis and put some potential of computer use and of modelling perspectives in focus of attention.

References 18 titles including:

Gravemeijer et al. (2002). *Symbolising, modeling and tool use in mathematics education*. Kluwer.

Kelly, A. & Lesh, R. (eds) (2000). *Research design in mathematics and science education*. LEA pp. 307-333.

A NON-STANDARD MATHEMATICS PROGRAM FOR K-12 TEACHERS

Patricia Baggett — New Mexico State University

Andrzej Ehrenfeucht — University of Colorado

At New Mexico State University in Las Cruces, New Mexico, USA, we offer a program in mathematics, started in 1995, for practicing and future teachers which attempts to provide mathematical knowledge that is at the same time both modern and useful. It significantly changes the mathematical content of teachers' mathematical education. It leans toward concrete applications and design and creation of artifacts, and uses calculator technology from the earliest grades. The program is not connected to any specific curriculum.

We offer six one-semester courses (and a seventh in August 2005) covering topics that teachers from kindergarten through high school can use. Each course has a central focus and can be taken at the graduate level (by practicing teachers) and at the undergraduate level (by students who are future teachers). The foci are: Arithmetic and Geometry (mainly for elementary teachers), Algebra with geometry and Use of technology (mainly for middle school teachers), and Mathematics with science, Algebra with geometry II, and Calculus with hands-on applications (mainly for high school teachers). Undergraduates act as apprentices to practicing teachers and are required to make at least ten hours of visits to their classrooms, where they observe, co-teach, and teach under their mentors' supervision. Teachers and future teachers often teach lessons that they studied in the university class to pupils, adapting them to their particular grade level. In the university class, writing is the central method used in assessing students' learning. We collect writings of teachers and undergraduates, and evaluate their understanding of the material, and how they taught the lessons in classrooms. We gather recalls of pupils who were taught the lessons, and artifacts that they created. So we can see what has been learned at several levels.

Participants consistently evaluate the program as relevant and interesting. We know that many alumnae and alumni who are now practicing teachers still use the lesson plans that they originally studied in these courses. We evaluate the effectiveness of individual lessons and courses, but not of the program as a whole.

In the poster we address three aspects of our program: What we are attempting to teach and why, and how it is being done. We will include examples of specific lesson plans, show samples of the work of pupils from different grades, and discuss evaluations of the lessons and courses.

Many lessons used in the courses, current syllabi, and a more complete description of our program, are at http://math.nmsu.edu/breakingaway.

2005. In Chick, H. L. & Vincent, J. L. (Eds.). *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 1, p. 302. Melbourne: PME.

SUPPORTING THE DEVELOPMENT OF MIDDLE SCHOOL MATHEMATICS TEACHERS'EVOLVING MODELS FOR THE TEACHING OF ALGEBRA

Betsy (Sandra E.) Berry

Purdue University, West Lafayette, Indiana, US

This study investigates the evolving instructional models in the daily practice of middle school teachers as they design, test, and revise reflection tools to guide their teaching of algebraic thinking and modelling.

Many middle school mathematics teachers equate the teaching of algebra with demonstrating procedures for symbol manipulation, simplifying algebraic expressions and solving and graphing linear, quadratic and more complex equations. In the US, most students' first experiences with algebra are in a traditional algebra course offered at the 7^{th}, 8^{th} or 9^{th} year. Rather than traditional symbol manipulation instruction, students at all levels should have opportunities to model a wide variety of phenomena mathematically, to represent, explore, and understand quantifiable relationships in multiple ways. In order for this learning change to take place in classrooms, teachers' instructional models of teaching must change (Doerr & Lesh, 2003). This study investigates those teaching models as they evolve in the daily practice of middle school teachers as they design, use, and revise reflection tools to guide their teaching of algebraic thinking and modelling at the middle school level.

The ideas offered in this poster presentation are preliminary results from a research project in progress. The aim of this study is to document and articulate the change and growth of teachers as they use their classroom practice as a learning environment for their teaching. It adopts a design experiment method (Brown, 1992) in which the participating teachers are designing, implementing and revising reflection tools for analysing their practice as they design learning environments for their students to learn a "new" algebra.

References

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EVALUATION OF SUGGESTED ITEMS IN PORTUGUESE MATHEMATICS TEXTBOOKS

Regina Bispo
Applied Psychology Institute (ISPA), Lisbon

Glória Ramalho
Applied Psychology Institute (ISPA), Lisbon
Educational Testing Institute (GAVE), Ministry of Education, Portugal

Textbooks play an important role in mathematics education. Very often, teachers rely on textbooks to implement mathematics curriculum which influences students' achievement. Thus, analysis of textbooks may help to understand student's mathematics performance. In mathematical textbooks, the suggested items reflect situations where students can potentially be actively involved in the learning process. In order to enhance student's mathematical literacy, tasks should be designed to trigger the use of different cognitive processes. This study focuses on the analysis of mathematical items and purports to be a contribution to the analysis of textbooks.

In this research study, the proposed items of two textbooks were analysed according to the OECD/PISA framework. They were 9th grade mathematics manuals reasonably popular among teachers. Three components were analysed: context – the part of the student's world in which the tasks are placed; mathematical content – mathematics "major domains", and competencies – mathematical processes that need to be activated to solve a real problem through the use of mathematics. The cognitive activities that competencies encompass are grouped into three competencies clusters: (1) Reproduction; (2) Connection; (3) Reflection.

From the 344 items analysed, 61% of them did not present a context. This means that de majority of the items did not provide a situation that could be a part of students life. With respect to mathematical content, the items are mainly included in the "Quantity" and "Space and Shape" categories. Data analysis also showed that 81.4% of the analysed items only require competencies encompassed in the reproduction cluster. These are items that lead students, predominantly, to select routine procedures and/or apply standard algorithms. Also, they mainly involve mainly familiar contexts, clearly defined questions and require only direct reasoning and literal interpretation of the results.

In conclusion, the analysis showed that in most cases items suggested in manuals do not have a real-world context and only lead to the reproduction of practiced knowledge. This type of problems does not give the opportunity to perceive mathematics as a way of understanding. Instead they lead to believe that doing and knowing mathematics means memorizing and applying a sequence of algorithms/rules correctly.

THE INVESTIGATION OF CONCEPTUAL CHANGE AND ARGUMENTATION IN MATHEMATICAL LEARNING

Yen-Ting Chen   Shian Leou

Chung Hwa College of Medical Technology   Kaohsiung Normal University

Following the conceptual transition of learning and teaching, the object of knowledge construction have to be developed by students through their teachers. Therefore, the target of mathematical learning is to emphasize understanding of mathematical knowledge rather than repetition from memory. This paper reports on the performance of three students in their first-grade of senior high school on tasks about integral number, involving questions on divisibility.

This paper was a qualitative research project. The first purpose of this study was to use Posner’s (1982) conceptual change model (CCM) to inquire how the three students make others’ conceptual ecology become unbalanced by their dialogues and to bring their conceptual change under the cooperative learning context. The second purpose of this study was to use the framework proposed by Toulmin (1958) to examine the three students’ argumentative performances. The collected information included the videos, coding data recording the process of the three students’ learning, and the individual student’s papers.

The main results were: Firstly, The three students would change their conceptual framework after their conceptual ecology became unbalanced through communicating, thinking and reasoning with each other. Secondly, the approach of the three students’ argumentation included visual experienced argumentation, using examples argumentation and formal theory argumentation.

This highlights that the teacher can and should construct a learning context in which students can think, participate in mathematically valid argumentation, and develop meaningful mathematical learning.

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TEACHING TIME BY PICTURE BOOKS FOR CHILDREN IN MATHEMATICS CLASS

Jing Chung

Dep. of Math. Edu., National Taipei Teachers College, Taiwan, R.O.C

The experience of time is by no means strange to children. However, length, weight, area, etc can be taught by suitable physical objects but time could not be. Since the current belief in mathematics teaching stresses connecting real life (NCTM, 2000) and horizontal mathematization (Freudental, 1991) reasonable to provide some concrete situations in teaching time. This is conformed with the ides of the Realistic Mathematics Education (RME), anything that can help children to image, to development a model of some thing up to a model for something else, is good.

Monroe, Orme, and Erickson (2002) said that there are general or highly specific situation to help learners build time concept in children literature. For example, Willians told how the Shelans worked in cotton field from sunrise till sunset in (Working Cotton) to develop time vocabulary, time quantity and the order of events. The researcher led a group of teachers to search out picture books to teach time for different grades.

We collected sixteen pictures books, ten of them were published in Chinese translation. The title of twelve books contained time terms such as *Sunshine*, *Spring is here*, *Tuesday*, …etc. We analysed each book and listed themes associated with time concept. For example, *Clocks and more clocks* is suitable for low grades to discuss the order of events, how to tell time and to sense the flowing of time. *The Grouchy Ladybug* is suitable for low and middle grades to discuss the order of events, how to telling time, am-pm, what is a day, and the periodicity of day. *All in a day* is suitable for middle and high grades to discuss 24 o'clock, what is a day, the periodicity of day, and the time zone and lapse. In the design of a teaching plan, we use the picture books in the three ways, to induce interesting, to develop concept and to extend or apply.

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THE OVER-RELIANCE ON LINEARITY: A STUDY ON ITS MANIFESTATIONS IN POPULAR PRESS

Dirk De Bock [1 2] Wim Van Dooren [1 3] Lieven Verschaffel [1]

[1]University of Leuven, [2] EHSAL, European Institute of Higher Education Brussels and [3]Research Assistant of the Fund for Scientific Research (F.W.O.) – Flanders; Belgium

At several places, the practical and research-oriented literature on mathematics education (and occasionally also the literature on science education) mentions students' tendency to illicitly rely on linearity in non-linear situations. Recently, numerous manifestations of students' overuse on linearity in diverse mathematical domains and at various educational levels were re-analysed by De Bock, Van Dooren, Janssens and Verschaffel (2004) in order to unravel the psychological and educational factors that are at the roots of the occurrence and persistence of this phenomenon. As a result, these authors found three (complementary) explanatory elements for students' overuse on linearity, namely (1) students' experiences in the mathematics classroom, (2) the intuitive, heuristic nature of the linear model, and (3) elements related to the specific mathematical problem situation in which the linear error occurs.

This poster shows the results of an ongoing study on the overuse of linearity in newspapers and popular magazines. Different manifestations are discussed and related to the explanatory factors unravelled by De Bock et al. (2004). Moreover, these manifestations are classified and commented from the perspective of the authors' intentions while consciously or unconsciously overusing linearity. This led us to three different categories: (1) manifestations clearly intended to mislead and manipulate the reader, (2) authors' deliberate choices to justifiably simplify a non-linear situation for his or her audience, and (3) examples in which the author was clearly unaware of the problematic use of linearity in the given situation.

After having illustrated and categorized different manifestations of the overuse of linearity, we discuss the usefulness of misleading (linear) representations in popular press for mathematics education. Is it desirable and feasible to design learning activities based on misleading or partial (linear) representations that regularly appear in newspapers and magazines? Can we learn students to disguise this type of information and can it contribute to educate them to become critical citizens? To what extent this is a more general educational goal or a specific goal for mathematics education?

Reference

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SOFTWARE FOR THE DEVELOPMENT OF MULTIPLICATIVE REASONING

Dmitri Droujkov Maria Droujkova

Natural Math, LLC North Carolina State University

Interactive multimedia tools can help children make images, mathematize actions, link formal and informal representations, and notice properties of systems. Software can support the growth of mathematical reasoning from qualitative, intuitive grounding.

We research and develop a suite of programs helping young children work in multiplicative environments and see the underlying algebraic structures (Carraher, Schliemann, & Brizuela, 2000). To support various learning actions, suite parts provide different levels of openness and direction.

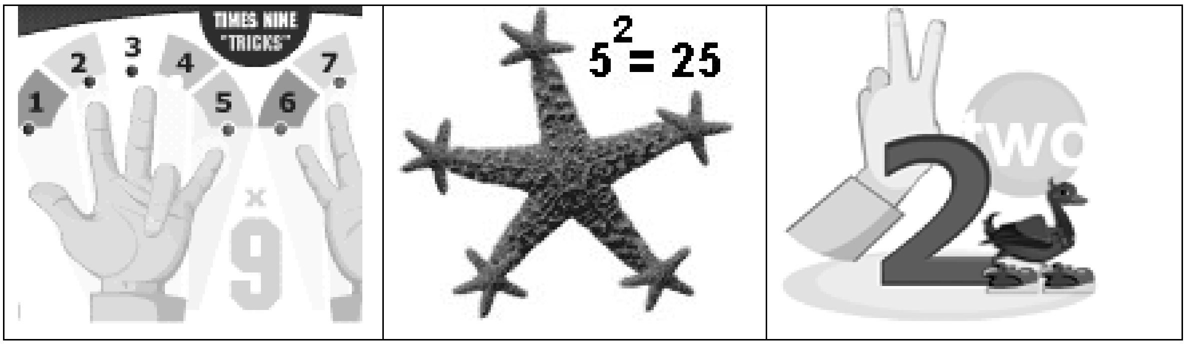

Figure 1: Screenshots of the software components

Theme playgrounds establish common mathematical actions, such as "finger calculator" tricks or creation of combination tables

Translation puzzles link different formal and informal representations and help children develop a mathematical language shared with others

Dynamic illustrations support interactive "eye openers" and grounding

Design worlds allow children to create their own representations

Problem solving tasks help with classical and novel multiplicative problems

The software helps children coordinate qualitative and quantitative worlds (Droujkova, 2004) in each context, providing qualitative grounding for mathematical reasoning.

References

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Droujkova, M. (2004). The spirit of four: Metaphors and models of number construction. Paper presented at the *26th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, Toronto, Ontario, Canada.

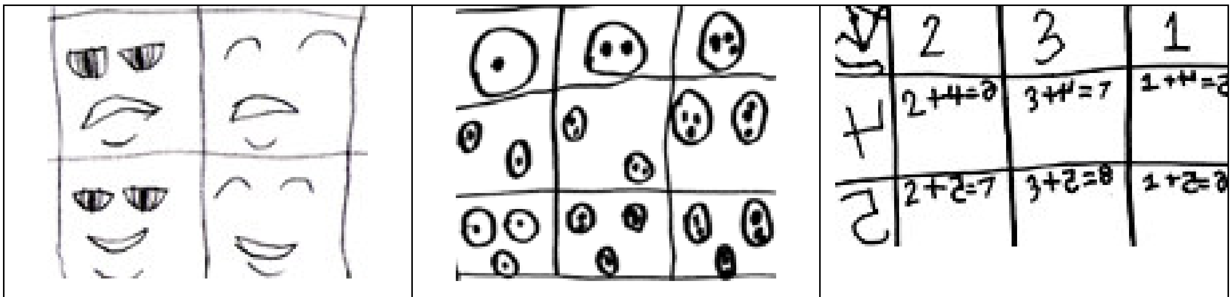
TABLES AND YOUNG CHILDREN'S ALGEBRAIC AND MULTIPLICATIVE REASONING

Maria Droujkova

North Carolina State University

Far beyond the humble role of a storage device, the table can be a powerful tool for young children's conceptual learning. Using tables in qualitative, additive and multiplicative worlds, children develop algebraic and multiplicative ideas such as covariation, binary operation, distribution, or commutativity.

This study focuses on children age four to seven working with table representations. Children start learning the row-column structure from the qualitative operation of *combining* features, such as eyes and mouths in simple face drawings. They move to iconic representations of quantities and *counting* operations, and to symbolic representations of numbers with *additive* (Brizuela & Lara-Roth, 2002) and *multiplicative* operations (Figure 1).



Combine mouths & eyes	*Count* dots & circles	*Add* numbers

Figure 1: Combining, counting, and adding operations in tables

Several issues with children's use of tables came up in the study. Children prefer to see features appear in each cell, rather than to use row and column labels. Children either work with a binary operation between co-varying row-column features, or with a unary operation on columns, varying the operation by rows. These two ways of thinking lead to significantly different table actions and reasoning. Children can transfer the table structure and actions between qualitative, additive and multiplicative worlds (Droujkova, 2004). Educators can help young children develop table reasoning qualitatively using established everyday ideas and transfer it to quantitative operations.

References

Brizuela, B. M., & Lara-Roth, S. (2002). Additive relations and function tables. *Journal of Mathematical Behavior*, 20(3), 309-319.

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STUDENTS' USE OF ICT TOOLS IN MATHEMATICS AND REASONS FOR THEIR CHOICES.

Anne Berit Fuglestad

Agder University College

This poster reports from a three year development and research project with mathematics classes in year 8 – 10. The aim was, in accordance with the curriculum guidelines (KUF, 1999), to develop and evaluate students' competence to chose appropriate ICT tool for a specific mathematical problem (Fuglestad, 2004). The project was situated within a social constructivist perspective of learning aiming to develop an ICT rich learning environments with opportunities for students' choices and discussions. In project meetings with the teachers every term some ideas and material for teaching were provided, and an important part was to report and discuss experiences, features of the ICT tools and further developments of ideas.

In a two weeks working period in the final part of the project the students were given a collection of 12 tasks to work on. The tasks were designed to give options for ICT use, with variation in levels and degree of openness; some had a clear question and others presented just an open situation and students had to set their own tasks. The students chose what tasks to work on, and what tools to use: mental calculation, a calculator, paper and pencil, ICT tools or a combination. They could work alone or in pairs and discuss their solutions. The work in the classes was observed, and partly audio and videotaped. Data was also collected in a questionnaire.

One or two weeks later the students were given a questionnaire connected to their experiences in the work, what tools they chose to use and why. They answered questions about tasks they had worked on, what they liked and did not like and for some new tasks they were asked to read and judge what they think were appropriate.

The results revealed that many students liked challenges and difficult tasks and disliked the same again and again. On the other hand some liked easy tasks, and overall students liked tasks they could master. The students gave reasonable answers concerning their choices of tools, for about 18 % their reasons were clearly related to features of the software, whereas for 46 – 60% less informative reasons were given.

The poster will display answers from the questionnaire and a selection of students' solutions to tasks and how the results relate to their choice of ICT tools.

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TEACHER ORIENTATIONS TO EQUIPMENT USE IN ELEMENTARY MATHEMATICS CLASSROOMS

Joanna Higgins

Victoria University of Wellington

Three orientations to equipment use in the classroom are examined in terms of the extent to which each supports students' thinking and discussion of mathematical ideas. The responsibility for action and the group configuration change across the three orientations of procedural, conceptual and dialogical. The comparison draws on excerpts from interviews and observations in three classrooms participating in the New Zealand Numeracy Development Project.

A COMPARATIVE ANALYSIS

The New Zealand Numeracy Development Project has emphasised the use of equipment through the introduction of a teaching model. The Teaching Model (Ministry of Education, 2004) drawing on the work of Pirie and Kieren (1989) represents levels of abstraction in representations of mathematical ideas. The model suggests that a process of working from using materials to using number properties be followed when encountering new mathematical ideas.

This poster uses a table format to compare three orientations to equipment use and illustrates each orientation with material drawn from interviews, classroom observations and project artifacts. The analysis of each orientation draws on activity theory (McDonald, Le, Higgins, & Podmore, 2004) to examine the claim that the Numeracy Development Project has shifted teachers' use of equipment from a focus on physical action on the equipment in a procedural orientation, to equipment used as a tool for thinking in a conceptual orientation, to equipment mediating discussion in a dialogical orientation.

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PROCESS OF CHANGE OF TEACHING ON RATIO AND PROPORTION BY MAKING AWARE OF A KNOWLEDGE ACQUISITION MODEL: CASE STUDY

Keiko Hino, Nara University of Education, Japan

Teaching and learning of ratio and proportion is a big issue in mathematics education because of its relevance in daily life but also for learning science and advanced mathematics. However, as shown by the results of several international achievement tests and nation-wide tests, the percentages of correct answers by Japanese children in ratio and proportion are not high, even though they often score highly on calculations. An assumption of this study is that this problem requires investigation and modification of everyday teaching practice in the mathematics classroom. In actual lessons, although teachers take account of correct instruction of textbook terms or notations, they do not necessarily recognize the relation between children's learning of them and the development of their proportional reasoning.

In this study, through a collaborative effort with a teacher in preparing, implementing, reflecting and revising lessons on ratio and proportion based on a model "mechanism of internalization of mathematical notations by learner" (Hino, 2002), opportunities are provided for the teacher with thinking about the relationship between teaching of terms, notations, calculations and/or formulas in the textbook on the one hand, and developing pupils' proportional reasoning on the other hand. The purpose of the study is to investigate the thinking process of the teacher when he faced a challenge in making aware of the model in his teaching.

In collecting data, we developed lesson plans on three content units on ratio and proportion based on the model. Over three months, the lessons conducted by the teacher were observed and behaviors of focused pupils were examined intensively. The teacher was informed of the results of the observation as early as possible. After every lesson, the teacher also reflected on his teaching and made a brief report about observations of children's thinking and notations. We also had time for a weekly discussion. The teacher was asked to say freely about his conflict, questions, worries, etc., and also creative ideas and inventions. In the poster, the teacher's thinking process is illustrated together with some episodes. An important theme is the emergence of a jointly-created perspective "transformation of pupils in the classroom." The teacher became interested in the pupil changes reported by the researcher. The perspective provided a situation of discussion between the researcher and the teacher and created ideas of teaching. Furthermore, a proposal of letting the pupils draw figures attracted the teacher's attention to overcome his worries, which also contributed to deepening the discussion between us.

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PRE-SERVICE MATH TEACHERS' BELIEFS IN TAIWAN

Hsieh, Ju-Shan

National Taiwan University of Arts

In Taiwan, to meet the educational reform of nine-years curriculum, it is urgent to change primary math teachers' instructional values and beliefs. There has been quite extensive research on this in Australia and the United States, but there has been less work in Taiwan and no work with pre-service teachers. There are two purposes proposed for the current work. The first is to develop the instrument into a stable measurement tool for considering teachers' self-beliefs in instructional approach. Second, because students need to spend three years to finish the primary teachers programs and they are from different learning background, it is necessary to explore whether students' grade levels, the variation in the departments, and the teaching background make differences in their teaching values.

I used two studies as a basis for my research, Clarke (1997) and Ross, McDougall, Hogaboam-Gray and LeSage (2003) and designed an instrument based on their frameworks with some items revised to meet the needs of instructional contexts in Taiwan. The questionnaire instrument considered the scope of the curriculum design, preparing open-ended activities, asking students to have multiple solutions, the use of discovery process to construct student's math knowledge, the role of math teachers as leaders, the use of manipulatives, student-student interaction, students' assessment, active teaching, and levitating student's confidence. A five-point Likert scale was used, from strongly agree to strongly disagree. Participants were pre-service teachers in the three-year primary school programs at National Taiwan University of Arts (NTUA) and 50 pre-service teachers were sampled for each grade. They completed the questionnaires, and the data were analysed using factor analysis and three-way analysis of variance.

Statistically significant differences among the groups of students involved in the courses were found for a number of items. Results depended on mathematical background and teaching experience. Specifically, students who take the math instruction course tend to help children find the answer, use different ways to solve problems, and connect other subjects to math and be able to prepare the math lessons. Those who have teaching experiences are more likely to use supplementary materials, use constructive approach, and lead students explain the answer.

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A PROCEDURAL MODEL FOR THE SOLUTION OF WORD PROBLEMS IN MATHEMATICS

Bat-Sheva Ilany & Bruria Margolin
Beit-Berl College & Levinsky College, Israel

In solving word problems in Mathematics one must create a bridge between mathematical language, which demands seeing the various mathematical components, and the natural language which itself demands a textual literacy. Identifying the components in the text depends on the meta-language awareness of the place of form, word or sentence in the text and especially awareness to symbols and syntax. Bridging between the natural language and the mathematical language needs a model that will connect the semantic situation and the mathematical form. This bridge will direct mental activity in finding possible solutions before a further deeper analysis of the problem (Greer, 1997). The literature available states that creating a model can be done in two different ways: translating the verbal situation into mathematical concepts (Polya cited in Reusser & Stebler, 1997) or alternatively organizing the mathematical content unit (Freudenthal, 1991). Our suggested procedural model shows how one can combine these two different ways.

We will demonstrate examples of mathematical word problems in which the solution depends on the transfer from a verbal situation to a mathematical form. We are suggesting a ten-stage model, which connects the verbal and mathematical languages. This model suggests an interactive multi stage process allowing decoding of the verbal and mathematical text in order to find the meanings of the word problem. This process of giving meaning according to the model suggested is one of creating a "textual world" based on the schema of the reader. This is formed by using a repetitive interactive action based on the following stages:

Decoding graphic symbols.
Understanding the obvious content.
Understanding the semantics of the problem.
Understanding the mathematical situation.
Making a correspondence between these two situations.
Matching the schema of the text and the schema of the reader.
Posing ideas for solutions.
Sieving out unsuitable solutions.
Making a mathematical representation.
Finding a solution which can be checked.

We will bring examples of using this model in solving word problems for the upper classes of primary school, high schools and teacher training. We will show that a process of stages using comprehensible schema with simple word problems will enable the pupil to confront more complex verbal problems.

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THE 5TH- 11TH GRADE STUDENTS' INFORMAL KNOWLEDGE OF SAMPLE AND SMAPLING

EunJeung Ji

Graduate School of Korea National University of Education

This paper investigated how well 5th- 11th grade 235 students recognize the concept of sample and sampling.

In the Korean curriculum, students learn the concept of sample, sampling and other concepts related to sample and sampling, when they have reached the 11th grade of high school. But before the 11th grade, they have an activity about data collection, data analysis and the formulation of conclusion. We then investigated and analyzed the informal knowledge of students before they receive formal instructions. The informal knowledge of students is very useful for later learning of statistics.

For this inquiry, I modified the content of MIC[1], the related concept of sample and sampling, and designed questions to inquire students' about informal understanding. The results enabled the identification of the maximum response rate for each question that each student agreed or disagreed with. In particular, it didn't agree with how students consider the characteristic of population in the process of sampling, and the students agreed on a sampling process without considering the characteristic of the population or the components that consist the population.

It showed that 5th grade students didn't investigate the data connected with sampling, and didn't understand the validity of sample survey process. While, 6th grade students equally understood sample size, sampling process, the reliance of data acquired through sample survey that applied to the source of judgment. But in details, it revealed that student had a misconception, or stayed at a subjective judgment level. The significant point is that many high school students didn't adequately understood a sample size with sampling.

Though statistics instruction has traditionally been delayed until upper secondary education, this inquiry convinced us that this delay is unnecessary as the Jacobs' result.

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FOSTERING TEACHERS' ETHNOMATHEMATICAL LEARNING AND TRAINING: HOW DONE IN FACT AND WHAT CAN BE LEARNED ABOUT?

Katsap Ada

Kaye College of Education

This poster presentation will report wherewith learning mathematics from a cultural perspective, or learning Ethnomathematics interpreted in college classroom environment, where student teachers, Jewish and Bedouin (an ethnic group of Arab background) alike, who came together in 'History of Mathematics' course, explored, learned and debated some basic-activities of mathematics in the cultural group they come from. The research, conducted during the course, was an attempt to examine the mathematical-socio-cultural dialogue on mathematics education that develops following the learning process. Further, it was an attempt to expose, from the teacher's perspective, the values that can emerge from introducing subjects identified with Ethnomathematics into the teachers' education. The methodological framework was based on Grounded Theory approach, which uses a comparative method for data analysis, when the data sources include lesson protocols, lesson plans, feedback-questionnaires and open interviews.

Ethnomathematics comprises a combination of the ethno, signifying the socio-cultural context, and mathematics, interpreted as corpora of knowledge derived from practices (D'Ambrosio 1985). Hence, etnomathematical training can direct teachers toward understanding that exposure to mathematics from practices helps to create a learning environment encouraging the links to the real social world (Katsap, 2004). Therefore, it is advisable to include Ethnomathematics in the pre-service mathematics education programs, where teachers are obliged to learn instructional skills, accommodate different backgrounds and understand that mathematics' values are a contribution by all (Shirley, 2001). The program of Ethnomathematics teaching in the course was designed in accordance with each culture, Jewish and Bedouin, and two mathematical themes, geometry patterns and time calculation, chosen as mathematical background, were applied to seven topics. Data samples of unique demonstrations made during the course will be provided at the poster.

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STUDENTS' MISCONCEPTION OF NEGATIVE NUMBERS: UNDERSTANDING OF CONCRETE, NUMBER LINE, AND FORMAL MODEL

Tadayuki Kishimoto

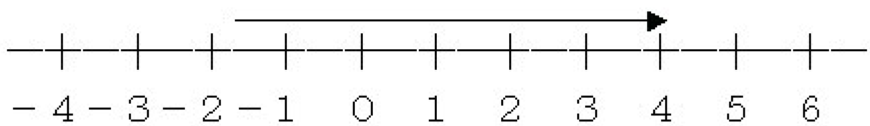
Toyama University

In Japan, it is difficult for many students to understand the operation with negative numbers. The previous researches (cf. Bruno and Martinon, 1996; Lytle, 1994) have been not enough to show why students have a misconception about negative numbers.

The purpose of this paper is to investigate why they have a misconception about negative numbers. Their understanding of negative numbers are analysed with regard to (1) concrete model (the east-west direction model), (2) number line model, and (3) formal model. And 129 students in seventh Grade were given some questionnaire tests. As a result, there became clear reason why they have a misconception about negative numbers.

(1) They keep on the conception formed through the informal experience. In calculating problem ((-1)-(-2)), they answered "-3" by doing (-1)+(-2). Because they said that result of operation would be less than (-1) if they subtract (-2) from (-1).

(2) They apply the mistake rule to relate the result of operation with the models. When some students were asked to represented the operation ((-2) ×(-3)) by using the arrow on the number line, they wrote as follows;



(3) They interpret the results of operation by the property involved references model. When they were asked to interpret the operation((-5)-(-3)) by concrete model, they said that "At first man walk at 5km to the west direction, and next at 3km, and now stay the west point from the start point at 8km". Because they said that they conjectured the operation (5+3=8) as "At first man walk at 5km to the east direction, and next at 3km, and now stay the east point from the start point at 8km".

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THE EFFECTS OF MATHEMATICS PROGRAM FOR GIRLS BASED ON FEMINIST PEDAGOGY

Oh Nam Kwon Jungsook Park Jeehyun Park Hyemi Oh Mi-Kyung Ju

Seoul National University Shilla University

The purpose of this research is to develop a mathematics program based on the feminist pedagogy (Jacobs, 1994; Warren, 1989) and to analyze its effects. 21 female students participated in this mathematics program for 3 weeks. All the participants finished the 9th grade to translate to the 10^{th} grade. The goals of this mathematics program are to entice young women to study mathematics and to convince their mathematical competence. Based on the feminist pedagogy, the program encouraged the participants to construct mathematics through social interaction based hand-on activities connected to experientially real contexts for girls.

The effect of this mathematics program was analyzed in mixed methods. We have collected video recordings of all class session, which were transcribe for discourse analysis. Tests were given to the students at the beginning and the end of the program in order to investigate comparatively the effect of the program on the students' conceptual understanding of function and data analysis. In addition, surveys and interviews were provided to inquire the students' affective change. Worksheets and reflective journals were collected to supplement the result of the data analysis.

The data analysis supported the significant impact of the program in the improvement of the students' conceptual understanding and affect toward mathematics. Specifically, the analysis of classroom discourse and tests showed that the students' mathematical reasoning has changed from analytic to holistic and from linear to nonlinear. This change is considered to reflect the development of the students' willingness to approach mathematics in diverse ways, which is one of the characteristics of good problem solver. Moreover, the analysis of interview and survey showed that the students became to realize their mathematical competence and the importance of social skills in doing mathematics through their participation in this program. These positive results suggest that further research is of essence to develop an inclusive instructional model for mathematical empowerment of female students.

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ARGUMENTATION AND GEOMETRIC PROOF CONSTRUCTION ON A DYNAMIC GEOMETRY ENVIRONMENT

Víctor Larios Osorio

Facultad de Ingeniería, Universidad Autónoma de Querétaro, México.

The proof is an important teaching object at secondary school, whose several functions that takes in mathematics education allow it to be the validation mean for the generated knowledge but also be one mean to communication, to discovering, to exploration and to explanation. However, its learning has several difficulties related with different aspects such its conception or meaning, the difference in Geometry between drawings and figures, and the relation between proof and argumentation.

To study this situation, we have planned a research project to study the arguments generated at geometrical proof's development in one secondary school at México, under a dynamic geometry environment (with Cabri-Géomètre), on the field of triangle and quadrilateral geometry, and considering some theoretical arguments that seems to us relevant, like the Cognitive Unit of Theorems (Boero et al., 1996) and the differences about proof's meaning among different institutions in Godino and collaborators' sense (see Godino & Batanero, 1994).

In this project is proposed that proof's meaning in scholastic institution is linked with argumentative actions in which is looked the conviction of individual and other people that some mathematical fact occurs, and that argumentation has a deductive structure.

We used activities with triangles and quadrilaterals for a teaching experiment, and we noted students' behaviours which show that figural and conceptual components (Fischbein, 1993) have not harmony, and appeared too confusions in the objects' meanings. Furthermore, the presence of argumentative justifications of observed properties and the apparent lack of a "natural" need to justify through mathematical proofs (deductions) in this educational level might lead us to re-expound the proof's meaning at educational context, both by teachers and by students, although this meaning must take as reference that of mathematicians' institution.

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WHAT'S WRONG WITH THIS SOLUTION PROCEDURE? ASKING CHILDREN TO IDENTIFY INCORRECT SOLUTIONS IN DIVISION-WITH-REMAINDER (DWR) PROBLEMS

Síntria Labres Lautert & Alina Galvão Spinillo

Federal University of Pernambuco, Brazil

Research studies in psychology and in mathematical education show that children make different kinds of mistakes when solving division-with-remainder (DWR) problems (e.g., Silver, Shapiro & Deutsch, 1993 and Squire & Bryant, 2002). As important as knowing children's difficulties is to examine whether they are able to identify mistakes when faced with incorrect procedures of solving division-with-remainder (DWR) problems. The present study aimed to investigate this aspect in 100 low-income Brazilian children who presented difficulties in solving this kind of problems at school. Half of these children formed an experimental group and the other half was the control group. Children in the experimental group individually received specific intervention involving the solution of division-with-remainder (DWR) problems (materials were made available), in which the examiner presented situations that required the child to (i) understand the effect of increasing/diminishing the divisor over the dividend; (ii) understand the inverse relations between the number of parts and the size of the parts in a division problem, and (iii) analyze correct and incorrect processes of solution. All the children were submitted to a pre-test and a post-test, both consisting of six incorrect procedures of resolution related to the same kind of mistakes that children usually make. In each situation two procedures of resolution were shown: one incorrect and another one correct. The children were asked to identify which of the two procedures of resolution was incorrect and to explain the nature of the mistake identified. The data were analyzed according to the number of correct responses and according to the explanations given. No significant differences were found between groups in the pre-test. However, in the post-test children in the experimental group were significantly more successful than those in the control group. These children performed significantly better in the post-test than in the pre-test. The main conclusion was that the intervention helped the children to identify and analyze the types of incorrect procedures of resolution, as well as to develop a metacognitive ability related to problem solving. This ability is crucial for the learning of mathematics.

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INTERVIEWING FOUNDATION PHASE TEACHERS TO ASSESS THEIR KNOWLEDGE ABOUT THE DEVELOPMENT OF CHILDREN'S EARLY NUMBER STRATEGIES

Ana Paula Lombard, Cally Kühne, Marja van den Heuvel-Panhuizen,

Cape Peninsula University of Technology , South Africa
University of Cape Town, South Africa
Freudenthal Institute, University of Utrecht, Netherlands

This poster addresses the tool that was used in the COCA (Count One Count All) project for assessing the teachers and the results thereof.

The purpose of the tool is a baseline assessment of the teachers' knowledge of early number strategies. After a two-year professional development programme, the tool will be used again to assess the efficiency of the intervention.

The professional development programme is connected to the Learning Pathway for Numeracy (LPN) that is being developed in the COCA project. This project is a SANPAD funded project carried out by the University of Cape Town (UCT) in collaboration with the Freudenthal Institute (FI), the Schools Development Unit (SDU) and the Cape Peninsula University of Technology (CPUT).

The data collection tool is a structured interview in which the teachers have to inform the interviewer about their knowledge of solving operations with numbers up to 100. The teacher is presented with a number of slips containing learner strategies and has to arrange them in an instructional sequence according to their classroom experience. Apart from some background information about the COCA project, the poster will show the tool that was used and a selection of the data that was collected with it. In addition to the results presented in text form on the poster, photographs and video clips will be shown on a laptop.

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A STUDY OF DEVELOPING PRACTICAL REASONING

Hsiu-Lan Ma

Ling Tung College, Taiwan

Problem solving and reasoning are two of the five process standards (NCTM, 2000). They are two important skills for students to cope with the real world. According to the results of TIMSS 2003, 4th graders in Taiwan did not do well on the reasoning problems; only 43.3% students passed. As a result, the researcher was drawn to study this phenomenon. This study, one of several researcher's projects via internet discussion board funded by National Science Council in Taiwan (e.g., Ma, 2004), will investigate and analyse the development of students' practical reasoning.

The participants in this study were 24 fifth graders from Taichung County, Taiwan, who had basic computer skills, used the internet regularly, and had computer and internet access at home. The participants were divided into 6 groups. Each group was given a theme, which included hiking, culture, food, historic spot, sightseeing, and picnic, and then were asked to plan a trip according to their themes. The main problems for these 5th graders to solve, for example, were: What do we need to do before our trip? How do we plan our budget? Participants had conversations on an internet discussion board, in order to preserve the problem-solving and reasoning processes. Each group worked on the project by communicating and exchanging ideas with others. The teacher applied the five-step heuristics (i.e., focus, analyse, resolve, validate, reflect), claimed by Krulik and Rudnick (1993), as the instruction program to guide the students to develop practical reasoning. In addition, she monitored the interactions among participants, and also kept them on track via the same discussion board. This activity lasted from October, 2003 to June, 2004.

Based on this study: (a) The researcher gained insights about how students generated practical reasoning, and applied the five-step heuristics of the reasoning with sub-skills. (b) These five heuristics were used back and forth when students settled on a situation through thematic approach. (c) The teacher played a critical role in this study, guiding students and helping groups to focus on the sub-topics related to their themes. By participating in this study, students applied their mathematical skills and knowledge to problem solving and reasoning for daily real-life situation.

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DEVELOPING IDEAS: A CASE STUDY ON TEACHING 'RATIO' IN SECONDARY SCHOOL

Christina Misailidou

University of Manchester

This poster provides results from a case study concerning the development of ideas for a more effective teaching of 'ratio' in secondary classrooms. Such ideas developed in three stages. The first stage involved teaching suggestions that were generated from the author's study of problem solving in small groups of pupils. The result of that study was a 'cultural teaching device', i.e., a combination of a challenging task context, a pictorial model and a related collection of arguments and teaching interventions: this device has been found to aid the pupils' proportional reasoning (Misailidou & Williams, 2004).

The second stage of development involved communicating the 'teaching device' to a 'teachers' inquiry group' ('TIG'): this was a group consisting of secondary mathematics teachers and researchers who met and worked together with the aim of developing effective teaching practice. After discussing and reflecting on the author's proposal, Alan, a teacher and member of the group decided to teach 'ratio' in his class. Thus, the third stage of development involved Alan's implementation of the teaching suggestions in his class. Alan, adopted the general principles of the teaching device but its particular aspects were 'transformed' to suit Alan's teaching style and the needs of his class: the task context was altered and the pictorial model was substituted by tabular arrangements.

This poster presents a 'model' of the development of effective teaching on ratio: the cultural teaching device as originally proposed by the author, the transformations through the TIG and Alan's particular needs and the final teaching device that was implemented in Alan's class. It is argued that such a 'model' is necessary for the successful implementation of a research proposal in a normal classroom.

Acknowledgement: The project was funded by ESRC (Award R42200034284).

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ANALYSES OF US AND JAPANESE STUDENTS' CORRECT AND INCORRECT RESPONSES: CASE OF RATIONAL NUMBERS

Yukari Okamoto

University of California Santa Barbara, USA

Bryan Moseley

Florida International University, USA

Junichi Ishida

Yokohama National University, Japan

International research has documented that US students lack solid understandings of rational numbers in comparisons to their peers in high performing nations such as Japan. As part of a study of US and Japanese students' and teachers' conceptual understandings of rational numbers, the present study examined students' solutions to part-whole, proportion and ratio problems. We were particularly interested in students' correct and incorrect responses that may help us uncover their rational number understandings. Data were collected from 183 fourth graders in Japan and 91 fourth graders in the US. Effort was made to recruit students so that achievement levels were comparable between the two nations. In each nation, students worked on a paper-and-pencil test that included multiplication and division problems, part-whole problems and word problems about proportions and ratios.

As expected, no national differences were found on the overall performance between the US and Japan, F (1, 274) = 1.00. The general patterns of performance were remarkably similar. On most problems, students' correct answers were expressed in one way. For two of the proportion problems, however, Japanese students came up with multiple ways to express correct answers. To figure out how many cups of water is needed to make a soup for 6 people when the recipe for 8 people calls for 2 cups of water, Japanese students responded with 1 1/2, 1 2/4, 3/2, and 8/6 cups in addition to the standard 1.5 cups. For the problem of the amount of cream for 6 people when the recipe for 8 calls for 1/2 cup, we saw .75/2 among Japanese responses. As for incorrect responses, we found that more US than Japanese students solved the soup problem by simply multiplying the recipe for 8 by 6 to find the amount needed for 6 people. On the ratio problems in which students were asked to determine how much food to give to fish according to their relative size, we found that more US than Japanese students ignored the ratio given but instead focused on the relative size (e.g., bigger and smaller) to arrive at an answer. We have recently collected additional data to examine if these cross-national characteristics can be replicated.

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MATHEMATICAL ACTIVITIES AND CONNECTIONS IN KOREAN ELEMENTARY MATHEMATICS

JeongSuk Pang

Korea National University of Education

In recent international comparisons Korean students have consistently demonstrated superior mathematics achievement not only in mathematical skills but also in problem solving (e.g., OECD, 2004). This draws attention to mathematics education in Korea (Grow-Maienza, Beal, & Randolph, 2003). A textbook is a strong determinant of what students have an opportunity to learn and what they do learn because all Korean elementary schools use the same mathematics textbooks and, more importantly, almost all teachers use them as their main instructional resources.

The most recently developed seventh curriculum and concomitant textbooks have a level-based differentiated structure and emphasize students' active learning activities in order to promote their mathematical power. The textbooks intend to provide students with a lot of opportunities to nurture their own self-directed learning and to improve their problem solving ability. This resulted from the repeated reflection that previous textbooks were rather skill-oriented and fragmentary in conjunction with the expository method of instruction.

Given this background, the poster presents main characteristics of current elementary mathematics textbooks along with some representative examples. The characteristics include encouraging students to participate in concrete mathematical activities, proposing key questions of stimulating mathematical reasoning or thinking, reflecting mathematical connections, and assessing students' performance in a play or game format.

With regard to each characteristic, this poster first presents some background information and rationales in brief. It then shows examples from the textbooks so as to highlight key features, followed by an elaboration on the examples. The topics of examples vary such as subtraction with base-10-blocks, rotation of a semicircle, calculation of a decimal divided by a fraction, and figuring out divisors. As for mathematical connections, in particular, this poster displays how the addition and subtraction of fractions with different denominators at a fifth grade level are based on other related concepts and operations at the previous grade levels.

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ONLINE INSTRUCTION FOR EQUATION SOLVING

Daphne Robson

Christchurch Polytechnic Institute of Technology/Lincoln University

Mathematical learning theory has been used as the basis of an interactive visual program with frequent feedback for learning equation solving. This was then used to investigate its effects on the approach students chose to solve linear equations.

BACKGROUND

Engineering and science students use many mathematical formulae which need to be transposed. A good understanding of equation solving is needed and as students often find it difficult, online instructional software was developed. Mathematics requires a particular type of thinking: the ability to see both the global strategic view of a problem as well as the detailed view of each step. This type of thinking, in the context of mathematics, has been called versatile thinking (Thomas, 1995).

SOFTWARE

An important feature of the software design was to separate the strategic and detailed thinking and allow students to develop strategies for equation solving without performing the detail of each step. This was achieved by providing frequent feedback of the type recommended by Tedick (1998). A sequence of screen shots of the software will be used on the poster to show its features.

METHOD AND ANALYSIS

A pre-test/post-test trial used adult students with the software recording their choices. This sequence of information showed the approach students chose to solve equations at each stage and this will be displayed on the poster.

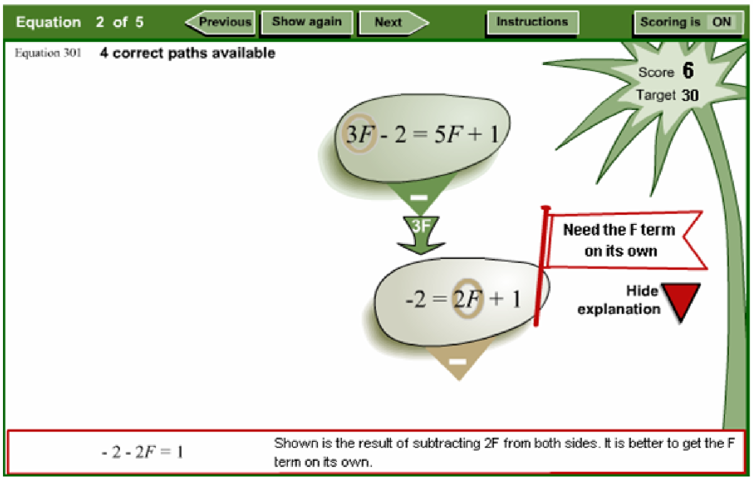

Figure 1: Screen shot of software

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USING THE CALCULATOR TO UNDERSTAND REMAINDERS OF DIVISIONS AND DECIMAL NUMBERS[2]

Ana Coêlho Vieira Selva & Rute Elizabete de Souza Rosa Borba
Universidade Federal de Pernambuco – Recife – Brasil

This study investigated the understanding of the meaning of remainders of divisions and their decimal representations through an intervention in which problems solved with pencil and paper were compared with the results obtained in the calculator. Vergnaud (1987) stressed the importance of using different symbolic representations whilst teaching, however, some forms have been given priority over others and very few usage of multiple representations has been observed at school (Selva, 1998).

Children aged 9 and 10 (n=18) and aged 11 and 12 (n=14) of a Brazilian state school took part in the study that involved a pre-test, an intervention and a post-test. In the intervention the children were assigned to one of two conditions: Group 1 – initially solved the problems with pencil and paper and then with the calculator; Group 2 – solved the problems initially with the calculator and then with pencil and paper.

It was observed that the mean score in the pre-test of the 9 and 10-year-olds was 53.71%. In the pos-test the mean score of the children from the paper-calculator group was 85.19% and of the calculator-paper group was 59.53%. The mean score of the 11 and 12-year-olds in the pre-test was 64.29%. In the post-test the mean score of the paper-calculator group was 90.48% and for the calculator-paper group it was 95.29%. Thus, the intervention was effective for both groups of 11 and 12-year-olds but for the 9 and 10-year-olds significant improvement was observed only in the condition in which the children used the calculator after solving the problems with pencil and paper. Possibly it was easier for these children to relate both representations (drawings or algorithms in paper and decimals in the calculator) when they first used a more familiar representation and reflected about the result obtained.

It was concluded that understanding decimal representation is not always straightforward but children can benefit from teaching conditions that promote the relations between this representation and more familiar ones. Interventions like the one proposed in this study can lead children to raise hypotheses about decimals and should be considered by teachers that teach mathematics.

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[2] This research was sponsored by FACEPE (Fundação de Amparo a Ciência e Tecnologia do Estado de Pernambuco).

GENERATIVE ACTIVITIES AND FUNCTION-BASED ALGEBRA

Walter M. Stroup Sarah M. Davis

University of Texas at Austin

This poster will display the materials and results of a semester long intervention with two Algebra 1 teachers. Researchers worked with teachers to create function-based generative algebraic activities which would engage all students and foster greater understanding of the key concepts of equivalence, equality and linear equations.

The multiple-strands based approach to curricula promoted by the NCTM has not impacted the role of the single-strand Algebra I course as gatekeeper in the US educational system. If anything, Algebra I is more central at many levels. In secondary mathematics, improving outcomes in Algebra I is, perhaps, the single most strongly felt need at nearly every level in the national educational system. Traditional approaches to improving outcomes (e.g., doubling class time) have had only minimal success. The No Child Left Behind legislation requires that we "raise the bar" of performance for all students and do so in a way that also closes the gaps in performance identified by disaggregating testing, enrolment and graduation outcomes.

We have very good evidence pointing to the effectiveness of function-based algebra at a small scale (Brawner, 2001) and at a very large scale (NCES, 2003). We need solid mid-level results that point specifically and in a detailed way to the effectiveness of function-based algebra. With the requirement that local, state and national adoptions need to be scientifically based, it is vital that research speak directly to this requirement. Additionally the research must be optimized to speak to issues of raising the standards of performance for all students. Generative activities utilize student created artefacts as the core for instruction. For example, using next-generation classroom networks, a possible generative activity is to have all students submit a point whose Y value is twice the X. This group created, set of points, displayed to the class, becomes the focus of discussion.

In collaboration with the Texas Educational Agency and Texas Instruments a semester long intervention was done in a rural high school, just outside of a major southwest city. The study used a Solomon 4 research design with 250 students in both treatment and control groups. Throughout the semester, researchers worked with teachers to create and implement a series of function-based, generative activities utilizing next-generation classroom networks.

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TEACHING STATISTICS WITH CONSTRUCTIVIST- BASE LEARNING METHOD TO DESCRIBE STUDENT ATTITUDES TOWARD STATISTICS

Yea-Ling Tsao

Taipei Municipal Teachers College

Department of Math Computer Science Education

In this study, the researcher examined the effect of a semester-long, constructivist-based learning approach method on student attitudes toward statistics in an introductory statistics course. A major goal of an introductory statistics class is to teach students to think critically, using the fundamental concepts of statistics. Students should be able to organize and summarize data, draw inferences from such summaries, and incorporate such summaries and inferences into reports. Constructivist-based learning techniques promote learning through small group work experiences and involvement in learning activities other than just listening. These can include projects that require class participation through hands-on experiments or demonstrations that illustrate lecture material.

The author investigated whether students who engaged in constructivist-based learning environment performed more positive attitudes toward statistics. In order to answer research question, a t-test was used to compare the average scores on four subscales of the Survey of Attitudes Toward Statistics (SATS) performance of pretest and posttest. The t-test results indicated that there was a statistically significant difference between the SATS mean score of the pretest and posttest, at the 0.05 significance level. Using $\alpha = 0.05$ as the pre-study determined level of testing, there was sufficient evidence to reject the null hypothesis regarding differences in the measure of the average scores on the SATS at the beginning and the end for the students learning statistics with constructive learning approach.

The findings reported in this article show that students who were exposed to constructivist-based learning approach in an introductory statistics class gained positive attitudes. These results suggest that such constructivist-based learning techniques may be useful for enhancing learning. Constructivist-based learning methods may also offer alternative learning opportunities for students who do not fully grasp course material in the traditional lecture format. Constructivist-based learning approach provides students with the opportunity to apply theory to real-life situations and bring concepts and theories to life, thereby enhancing student learning.

STUDENT BEHAVIORS THAT CONTRIBUTE TO THE GROWTH OF MATHEMATICAL UNDERSTANDING

Lisa B. Warner
Rutgers University

The Pirie-Kieren model for the growth of understanding (Pirie & Kieren, 1994) provides a framework for analyzing student growth in understanding, via a number of layers through which students move, both forward and backward. The Pirie-Kieren model for the growth of understanding was conceived as a dynamic model, in which student movement between layers is a critical feature. Yet in the model itself there is no indication of the force, or motive, that impels a student to move from one layer to another. This poster documents a model for a motive that stimulates moves through the layers that form the structural base of the Pirie-Kieren model. These student behaviors involve a change in the learner's focus of attention (e.g., Warner, Alcock, Coppolo & Davis, 2003) and include the ability of the learner to: interpret his/her own or someone else's idea (through explaining, questioning and/or using it; reorganizing and/or building on it); use multiple representations for the same idea; link representations to each other; connect contexts; raise hypothetical situations based on an existing problem (such as a "What if" scenario).

In this poster, an example of three sixth grade students' movement through the layers is used to identify how the behaviors relate both to each other and to the overall process of understanding (movement through the layers in the Pirie-Kieren model). A summary of the percentages of observed student behaviors that were associated with a move to a particular layer in the Pirie-Kieren model are also displayed.

Results indicate that for each of the three students, certain behaviors, including student questioning, explaining, re-explaining and using of one's own or others' ideas, in the inner layers of understanding, appeared to stimulate a move to setting up hypothetical situations, connecting contexts and linking representations to each other, which is associated with moves to outer layers of understanding. In the outer layers of understanding, setting up hypothetical situations also appeared to stimulate a move to connecting contexts and linking representations, which is associated with moves to the outer-most layers of understanding.

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ASSESSMENT OF SPATIAL TASKS OF GRADE 4-6 STUDENTS[3]

Helena Wessels — Laerskool Lynnwood, South Africa
Dirk Wessels — University of South Africa

The importance of geometry and the development of spatial abilities for literacy, especially mathematical literacy, now is an excepted fact. Different authors concur that spatial perception does not consist of a single skill or ability (Tartre 1984; Del Grande 1987, 1990). Del Grande (1987: 127; 1990:14) describes seven spatial abilities: eye-motor coordination, figure ground perception, perceptual constancy, position-in-space perception, perception of spatial relationships, visual discrimination and visual memory. These seven spatial abilities can be grouped into two major categories, i.e., spatial orientation and spatial visualization (Tartre 1990: 217). Teachers are not aware of the fact that researchers consider proper and effective spatial development of the young learner more complex than number development (Bryant 1992: 7; Van Niekerk 1997; Wessels 1989), therefore spatial development is often neglected in the teaching and learning of geometry.

An instrument consisting of 23 spatial tasks was developed to study learner responses to spatial problems in order to determine the present developmental state of spatial abilities and the efficacy of current teaching strategies used in a government school in South Africa. The degree of difficulty of the spatial tasks was determined using the Wattanawaha's classification system (Clements 1983:16) using four independent properties to classify spatial problems. Assessment rubrics for each task were set coding answers according to a nominal scale ranging from 0 to 1 or 2, depending on the requirements for the task. Examples of the coding of difficulty of the tasks, assessment rubrics and the coding of learner responses will be given, as well as preliminary findings of the research.

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DIDACTICAL ANALYSIS OF LEARNING ACTIVITIES ON DECIMALS FOR INDONESIAN PRESERVICE TEACHERS

Wanty Widjaja

University of Melbourne, Australia

Research (Stacey et al., 2001) has indicated that misconceptions and difficulties in understanding decimals persist with preservice teachers. It is posited that many Indonesian preservice teachers will also have limited conceptual knowledge on decimals as the approach of teaching and learning decimals is dominant with the exposure of whole numbers rules in particular when dealing with addition and subtraction of decimals.

A set of learning activities on decimal notation adapted using 'theory guided bricolage' approach (Gravemeijer, 1994, 1998) for Indonesian preservice teachers will be presented in a poster. The basic principles of *Realistic Mathematics Education* (RME) are employed in designing the learning activities in line with the current reform effort called "PMRI" to improve mathematics education in primary school since late 2001.

The concrete model based on length is employed in measurement context to promote an understanding of the repeated decimating process and give meaningful interpretation of place value in decimal numbers. This is in line with the didactical phenomenology principle where the application serves as a possible source of learning. The guided reinvention principle of RME is exercised in the activities where the preservice teachers use the concrete model in a measurement context and to observe the decimal relationships between different pieces of the model. Activities of constructing a decimal number in different ways using the concrete models are devised to explore the additive and multiplicative structures of decimals.

Didactical analysis will be presented in the poster to point out the possible contribution of the learning activities and how the learning tasks are related to basic tenets of RME.

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WORKSHOP ON DESIGNING "SCHOOL-BASED" MATHEMATICS INSTRUCTIONAL MODULES

Ru-Fen Yao

National Chia-Yi University, Taiwan

The main purpose of this workshop was to assist pre-service elementary teachers' in instructional design through providing them with opportunities for designing "school-based" mathematics teaching modules. The reason for the focus on "school-based" was the current curriculum reform in Taiwan. Theoretical foundations including "scaffolding theorem", "constructivism-based teaching strategies" and "the settings arrangement of cooperative learning" was applied in this ten-week workshop, researcher guided pre-service teachers to develop "school-based" mathematics instructional modules step by step. There were 41 undergraduates at 3rd and 4th grades in a national university of southern Taiwan participating in this workshop. Students were initially divided into 8 groups according to their choice, 4-6 students in a group. The issue of "school-based curriculum development" was emphasized by offering pre-service teachers many practice opportunities of instructional design. As for the design of "school-based" teaching module, every group firstly chose an elementary school as the basis of developing school-based curriculum. By collecting information from Internet, libraries, or interview with the elementary school, group members could understand the background, feature, and resource of this school. After coordinating with the mathematic teaching materials in elementary schools, the draft of "school-based" teaching module was developed. The researcher guided pre-service teachers to reflect, to examine and to revise their designs by cooperation, sharing, and discussion within and between teams. Eight "school-based" mathematics instructional modules were developed (see Table 1). From the process of developing instructional modules of pre-service teachers, the researcher frequently reflected on the contents and methods of teacher-preparing, and tried to find the important components and appropriate way to prepare pre-service teachers' in mathematics instructional design. The results showed that through a series of stages, "preparing stage, base stage, practicing stage, sharing stage, integration stage", it was useful for helping pre-service teachers to develop school-based teaching modules and enhancing their professional development in mathematics instructional design.

Table1. The list of the school-based instructional module design

Topic of the module	Mathematics-related concepts involved	Grade
Welcome to Ming-Hsiung	Multiplication and division; length; capacity; bar chart; three-dimensional pictures.	3
"An-Ping" vs. "Ping-An"	Addition and subtraction; multiple; length; time; the "space"-related concepts.	3
A visit in the frontline.	Addition, subtraction and multiplication; divide equally; multiple; length; weight; time; statistical table; bar chart.	3
A nice trip in "Peng-Hu"	Time; weight; two and three dimensional shapes; direction; bar charts.	5
A legend in Dong-Dan	Integer; average; addition and subtraction; time; hour; direction; perpendicular and parallelism; a cube in the shape of a rectangle; a prism in the shape of a triangle.	5
A trip in Bei-Dou	Sale; direction; bar chart; line chart.	4 & 5
The beautiful scenery in Ken-Ding	Length; time; scale; bar chart.	6
A "Green Island" melody	Length; capacity; square measure; volume; direction; scale; bar-chart.	6

CHILDREN'S "EVERYDAY CONCEPTS OF FRACTIONS" BASED ON VYGOSTKY'S THEORY: BEFORE AND AFTER FRACTION LESSONS

Kaori Yoshida

Research Fellow of the Japan Society for the Promotion of Science

Vygotsky (1934/1987) categorized concepts into two types: everyday concepts and scientific concepts. Everyday concepts are not based on a system but in rich daily contexts, thus sometimes children use them incorrectly from mathematical view points. In contrast, scientific concepts, (henceforth mathematical concepts) are based on "formal, logical, and decontextualized structures" (Kozulin, 1990). In accordance with Vygotsky's theory, Yoshida (2000; 2004) suggests these two concepts are finally sublated, i.e. everyday and mathematical concepts 1) conflict with each other through formal learning, 2) are lifted to higher levels respectively, and 3) are preserved as a unified concept, or sublated concepts.

It is difficult for children to understand fractions and worldwide researchers have struggled with more effective ways of learning/ teaching fractions. Based on the theory above, first, it is important to identify children's everyday concepts of fractions, and then it becomes possible to consider better learning/ teaching situations of fractions, taking the children's everyday concepts into consideration. I conducted pre- and post- questionnaire survey, before and after fraction lessons, on February 16, 2001 through an all-at-once style of exam in a classroom, and on March 15-18, 2001 as homework, respectively, targeting about 40 third graders making up one classroom in Japan. It was the first time for them to take fraction lessons formally in school at that time. (The lessons are discussed in detail in Yoshida (2004).

This poster will show the questionnaire entries with figures and children's answers with illustrations presented in the questionnaires, chart the questionnaire data, and identify what kind of "everyday concepts of fractions" children have and how the everyday concepts of fractions develop or do not develop through the lessons.

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